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Econ 201A - Riley
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Microeconomics - HW #3

1. Robinson Crusoe Economy

   (a) Assuming free disposal and $y_2 > 0$ (which seem to be necessary for the desired equivalence), then a production vector y is feasible if and only if

$$\begin{aligned} & y_2 \leq (L-a)^{1/2} = (-y_1 - a)^{1/2} \\ \Leftrightarrow \quad & y_2^2 \leq -y_1 - a \\ \Leftrightarrow \quad & y_2^2 + y_1 + a \leq 0 \end{aligned}$$

   (b)

$$\begin{aligned} U(x) &= x_1 + 6x_2 - \frac{1}{2}x_2^2 \\ &= \omega_1 + y_1 + 6y_2 - \frac{1}{2}y_2^2 \\ &\leq \omega_1 - y_2^2 - a + 6y_2 - \frac{1}{2}y_2^2 \\ \Rightarrow U(y_2) &= 24 - a + 6y_2 - \frac{3}{2}y_2^2 \end{aligned}$$

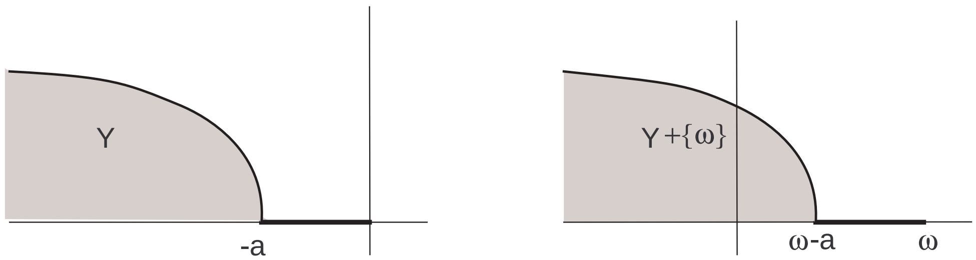
   Note U is concave in y_2, and

$$\begin{aligned} & U'(y_2) = 6 - 3y_2 && \Rightarrow y_2^* = 2 \\ & L = -y_1 \geq y_2^2 + a && \Rightarrow L^* = 4 + a \end{aligned}$$

   (c) In deciding whether to produce, Robinson compares $U(y_2^*)$ to the utility of consuming his endowment:

$$\begin{aligned} U(\omega) &= U(24, 0) = 24 \\ U(y_2^*) &= 24 - a + 6 \cdot 2 - \frac{3}{2} \cdot 2^2 = 30 - a \end{aligned}$$

   So it's optimal to produce when $a < 6$.



(d) For $a = 0$, Y is convex. Since U is quasiconcave, the indifference sets are also convex, so there exists an indifference set tangent to Y. The tangent slope is the (negative) equilibrium price. So

$$\frac{p_1}{p_2} = \frac{\partial y_2}{\partial y_1} = -\frac{U_1}{U_2}$$

Evaluating each of these, we get

$$\begin{aligned} \frac{\partial y_2}{\partial y_1} &= \left(\frac{\partial y_1}{\partial y_2}\right)^{-1} = -\frac{1}{2y_2} \\ -\frac{U_1}{U_2} &= -\frac{1}{6 - y_2} \end{aligned}$$

Combining and applying the ratio rule yields

$$\begin{aligned} &\Rightarrow \frac{p_1}{p_2} = \frac{1}{2y_2} = \frac{2}{12 - 2y_2} = \frac{3}{12} = \frac{1}{4} \\ &\Rightarrow y_2^* = 2, y_1^* = -4 \end{aligned}$$

Since only the price *ratio* can be uniquely determined, let's take $p = (1, 4)$. This yields optimal profits

$$\Pi^* = p \cdot y^* = -4 + 8 = 4$$

(e) As we've already shown, the consumer maximizes his utility by producing whenever $a < 6$. Applying the same analysis to the firm,

$$\Pi(y_2^*) = p \cdot y^* = 1 \cdot (-4 - a) + 4 \cdot 2 = 4 - a$$

So for $a < 4$, both parties profit by producing, and there is a Walrasian Equilibrium. For $a \in (4, 6)$ it is optimal to produce, but profits are negative.

(f) Given labor price $p_1 = 1$, the cost function is

$$C(y_2) = 1 \cdot L = y_1 = y_2^2 + a$$

Differentiating, we get $MC = 2y_2$, so if we force $p_2 = MC$ and give a lump-sum subsidy of s, then

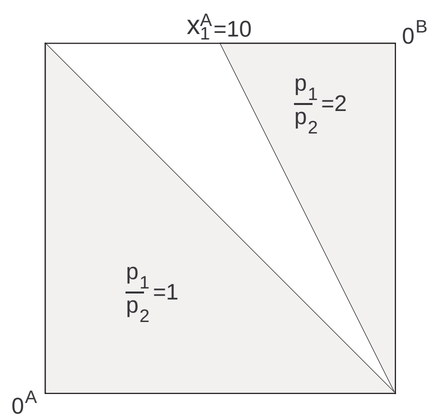
$$\Pi(y_2) = p_2 y_2 - C(y_2) + s = 4y_2 - y_2^2 - a + s$$

As before, this function is maximized at $y_2 = 2$, so

$$\Pi = 4 - a + s$$

If we want the firm to produce, we must offer a subsidy $s \geq a - 4$. Note it is not necessary to offer a per-unit subsidy, since if the firm opts to produce at all, it maximizes profits at y_2^*.

2. Edgeworth Box



(a) At all interior points, $MRS_A = 2 > 1 = MRS_B$, so there are no interior equilibria. Alex always has a greater preference for commodity 1, so at any price, someone will want to trade. Specifically, for price ratios between 1 and 2, Alex will wish to buy commodity 1, and Bev will wish to buy commodity 2.

Along the South and East edges of the box, however, this trade is impossible, as one or both of the parties is constrained. Specifically, on the East side, Bev is constrained by $x_1^B = 0$, so equilibrium prices determined by Alex's indifference curves. That is,

$$\frac{p_1}{p_2} = MRS_A = 2$$

(b) By the same analysis, on the South side of the box,

$$\frac{p_1}{p_2} = MRS_B = 1$$

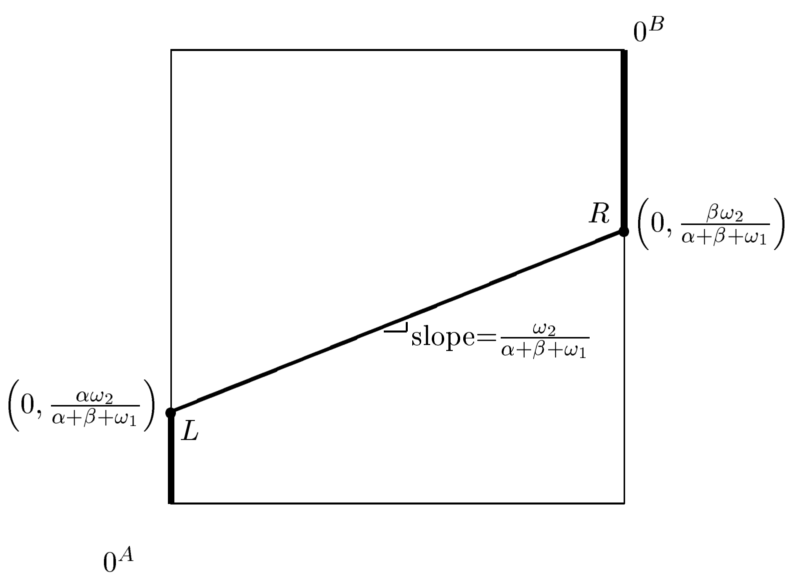
Both here and in (a), we can extrapolate from the WE points to determine the starting endowments that can result in these equilibrium allocations.

(c) As seen graphically, $\omega^A = (10, 16)$ does not lead to an equilibrium with price ratio 1 or 2. The only remaining equilibrium allocation is at $x^A = (20, 0)$, where both parties are constrained, so any price between 1 and 2 is an equilibrium. Therefore, the price ratio is whatever brings this endowment to that allocation:

$$\frac{p_1}{p_2} = -\frac{\Delta x_2}{\Delta x_1} = -\frac{-16}{10} = \frac{8}{5}$$

(d) Since both agents are constrained, any price ratio between 1 and 2 is an equilibrium.

3. Equilibrium and Pareto Efficiency



(a) At the interior solutions,

$$\begin{aligned}\frac{p_1}{p_2} &= \frac{U_1^A}{U_2^A} = \frac{U_1^B}{U_2^B} \\ &= \frac{x_2^A}{\alpha + x_1^B} = \frac{x_2^B}{\beta + x_1^B} \\ &= \frac{\omega_2}{\alpha + \beta + \omega_1}\end{aligned}$$

Note x_1^A and x_2^A are linearly related, so the contract curve is linear. At point L, where the curve hits the West side,

$$MRS_B(\omega_1, \omega_2) = \frac{x_2^B}{\beta + \omega_1} = \frac{\omega_2}{\alpha + \beta + \omega_1}$$

Solving for x_2^A at L,

$$x_2^A = 1 - x_2^B = 1 - \frac{\omega_2}{\alpha + \beta + \omega_1}(\beta + \omega_1) = \frac{\alpha\omega_2}{\alpha + \beta + \omega_1}$$

And symmetrically at the intersect point R,

$$x_2^B = \frac{\beta\omega_2}{\alpha + \beta + \omega_1}$$

(b) At interior solutions, price is constant, as shown in (a). On the West side, prices vary continuously from L to 0^A. At 0^A,

$$MRS_B = \frac{U_1(\omega_1)}{U_2(\omega_2)} = \frac{\omega_2}{\beta + \omega_1}$$

The symmetric result holds on the East side. Recalling that $\beta > \alpha$, the price ratio is largest at 0^B, so

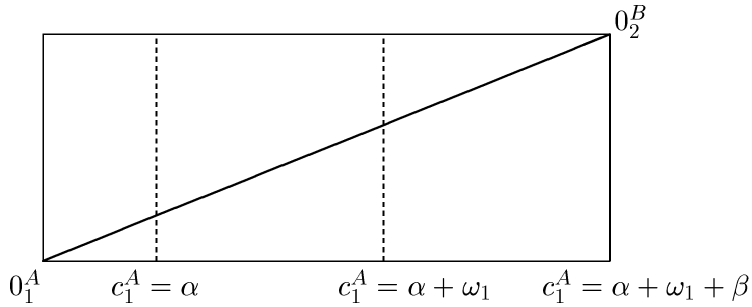
$$\frac{p_1}{p_2} \in \left[\frac{\omega_2}{\alpha + \beta + \omega_1}, \frac{\omega_2}{\alpha + \omega_1}\right]$$

(c) - (d) With this definition of c^A and c^B, we may write

$$U^A(c^A) = \ln(c_1^A) + \ln(c_2^A)$$

$$U^B(c^B) = \ln(c_1^B) + \ln(c_2^B)$$

Since these utility functions are symmetric, the contract curve would normally run down the diagonal of the box. But because part of the endowment is untradable, allocations outside the dotted lines are infeasible.



4. Edgeworth Paradox

(a) Assuming an interior solution, Alex's demand function is determined by her tangency constraint,

$$\frac{p_1}{p_2} = \frac{3(x_2^A - 3)}{x_1^A - 3}$$

along with her budget constraint, $p \cdot x^h = p \cdot \omega^h$, which can be rewritten

$$\frac{p_1}{p_2} = \frac{\omega_2^h - x_2^h}{x_1^h - \omega_1^h}$$

Combining these expressions, we get

$$\frac{p_1}{p_2} = \frac{3(x_2^A - 3)}{x_1^A - 3} = \frac{(2+a) - x_2^A}{x_1^A - (6-a)}$$

Multiplying the right side by $\frac{3}{3}$ and using the ratio rule, we get

$$\frac{p_1}{p_2} = \frac{3x_2^A - 9 - 3x_2^A + 6 + 3a}{x_1^A - 3 + 3x_1^A - 18 + 3a} = \frac{3a - 3}{4x_1^A - 21 + 3a}$$

$$\Rightarrow \qquad 4x_1^A - 21 + 3a = (3a - 3)\frac{p_2}{p_1}$$

$$\Rightarrow \qquad x_1^A = \tfrac{1}{4}\left[21 - 3a + \frac{p_2}{p_1}(3a - 3)\right]$$

The same analysis for Bev gives

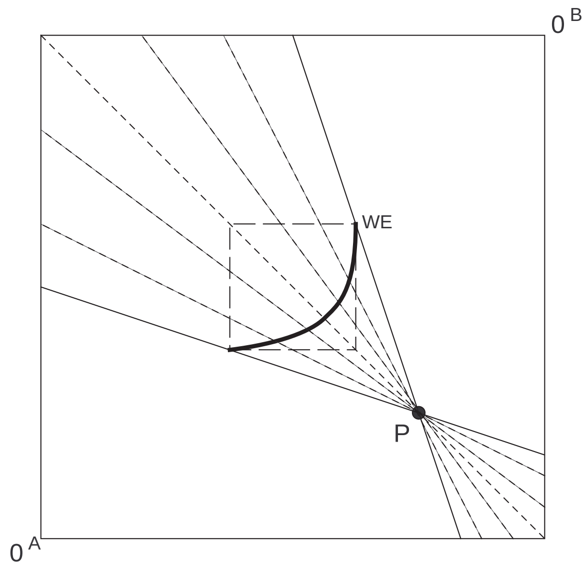
$$
\begin{aligned}
\frac{p_1}{p_2} = \frac{x_2^B - 3}{3(x_1^B - 3)} &= \frac{(6-a) - x_2^B}{x_1^B - (2+a)} \\
\Rightarrow \qquad \frac{p_1}{p_2} &= \frac{3-a}{4x_1^B - 11 - a} \\
\Rightarrow \qquad 4x_1^B - 11 - a &= \frac{p_2}{p_1}(3-a) \\
\Rightarrow \qquad x_1^B &= \tfrac{1}{4}\left[11 + a + \frac{p_2}{p_1}(3-a)\right]
\end{aligned}
$$

(b) Adding the previous expressions,

$$
\begin{aligned}
x_1^A + x_1^B &= \frac{1}{4}\left[\frac{p_2}{p_1}(3a - 3 + 3 - a) + 32 - 2a\right] \\
&= \frac{1}{4}\left[\frac{p_2}{p_1}2a + 32 - 2a\right] \\
&= \frac{a}{2}\left[\frac{p_2}{p_1} - 1\right] + 8
\end{aligned}
$$

The market clears when $x_1^A + x_1^B = \omega_1 = 8$, so with a price ratio of 1, any value of a will do.

(c) For $a < 0$, Alex's demand for good 1 is increasing in p_1, which certainly seems screwy.



(d) With $a = 0$, $\omega^A = P$ and the market clears at any price ratio, as seen graphically above.

(e) Since each agent's consumption set is $\{x : x_1, x_2 \geq 3\}$, only the points inside the dotted box can be equilibria. Consequently, any endowment outside the range of the price lines emanating from $P = (6, 2)$ cannot yield an equilibrium.