A guide to problem 1a

1. Take logs to get an expression for m_{t+1}.
2. Add r_{t+1} to each side.
3. Substitute the equations for Δc_{t+1} and r_{t+1} into the Right hand side.
4. Substitute the conjectured forms of wc_t and wc_{t+1}.
5. Substitute the expressions for x_{t+1} and σ^2_{t+1}.

You should now have an expression for $m_{t+1} + r_{t+1}$ that contains all non-random variables (subscript t) except for (η, e, w), which are independent with mean zero and variance one. This makes it easy to calculate the mean and variance of $m_{t+1} + r_{t+1}$.

Note that Hanno's Equation (5) can be written

$$E(m_{t+1} + r_{t+1}) + \frac{1}{2} Var(m_{t+1} + r_{t+1}) = 0$$

Find this and collect terms in x_t and $(\sigma_t^2 - \bar{\sigma}^2)$. The coefficients of both of these must be zero, so we get simultaneous equations,

$$\begin{aligned}
0 &= \ln\beta + \kappa_0 + (1-\rho)\mu + \frac{1}{2}\theta(1-\rho)^2\bar{\sigma}^2 \\
&\quad + (1-\kappa_1)A_0 + \frac{1}{2}\theta A_1^2 \varphi_e^2 \bar{\sigma}^2 + \frac{1}{2}\theta A_2^2 \\
0 &= 1 - \rho + A_1(\rho_x - \kappa_1) \\
0 &= \frac{1}{2}\theta(1-\rho)^2 + \frac{1}{2}\theta A_1^2 \varphi_e^2 + A_2(\nu - \kappa_1)
\end{aligned}$$

Solving these gives

$$\begin{aligned}
A_1 &= \frac{\rho - 1}{\rho_x - \kappa_1} \\
A_2 &= \frac{\theta(1-\rho)^2}{2(\kappa_1 - \nu)} \left(1 + \frac{\varphi_e^2}{(\rho_x - \kappa_1)^2}\right) \\
A_0 &= \frac{1}{\kappa_1 - 1}\left[\ln\beta + \kappa_0 + (1-\rho)\mu + \frac{\theta}{2}(1-\rho)^2\bar{\sigma}^2 + \frac{\theta}{2}\varphi_e^2\bar{\sigma}^2 A_1^2 + \frac{\theta}{2}A_2^2\right]
\end{aligned}$$