Andrew Iannaccone
Diego Ubfal

GAME THEORY - HW #1

1. **1A** $\succ$ **1B**, which we may rewrite as

$$.89(\$1\text{M}) + .11(\$1\text{M}) \succ .89(\$1\text{M}) + .10(\$5\text{M})$$

Assuming independence, this implies

$$.11(\$1\text{M}) \succ .10(\$5\text{M})$$

but this contradicts the preference **2B** $\succ$ **2A**. Thus, the empirical preferences violate the Independence Axiom.

2. Suppose there are subjective probabilities $\{r, g, b\}$. Then the third preference relation says

$$g(\$10) + b(\$10) \succ r(\$10) + b(\$10)$$

Assuming independence, this implies

$$g(\$10) \succ r(\$10)$$

but this contradicts the preference **2A** $\succ$ **2B**. Thus, the empirical prefences violate the Independence Axiom.

3. (a) We can simplify $U(\cdot)$ in the two regions:

$$\begin{aligned} z > y: \quad & U(x,y,z) = \pi u(x) + (1-\pi)[\alpha u(y) + (1-\alpha)u(z)] \\ y > z: \quad & U(x,y,z) = \pi u(x) + (1-\pi)[\alpha u(z) + (1-\alpha)u(y)] \end{aligned}$$

If we differentiate holding x and I constant, then

$$\frac{\partial z}{\partial y} = -\frac{p_y}{p_z}$$

Around a point where $y = z$, the differential is not defined, as the left and right limits are different:

$$\begin{aligned} \frac{\partial U}{\partial y^+} &= (1-\pi)\left[\alpha u'(y) - (1-\alpha)u'(z)\frac{p_y}{p_z}\right] \\ \frac{\partial U}{\partial y^-} &= (1-\pi)\left[-\alpha u'(z)\frac{p_y}{p_z} - (1-\alpha)u'(y)\right] \end{aligned}$$

The first-order condition is not therefore $\frac{\partial U}{\partial y} = 0$, but rather

$$\frac{\partial V}{\partial y^+} > 0 \quad \text{and} \quad \frac{\partial V}{\partial y^-} < 0$$

These become

$$\frac{p_y}{p_z} - \left(\frac{\alpha}{1-\alpha}\right)\frac{u'(y)}{u'(z)} > 0$$
$$-\frac{p_y}{p_z} + \left(\frac{1-\alpha}{\alpha}\right)\frac{u'(y)}{u'(z)} < 0$$

Since $y = z$, we have $u'(y) = u'(z)$, so the equations simplify to

$$\frac{1-\alpha}{\alpha} < \frac{p_y}{p_z} < \frac{\alpha}{1-\alpha}$$

(b) .

4. (a) Bayes Theorem says

$$\begin{aligned} P(D|T) &= \frac{P(D \cap T)}{P(D \cap T) + P(D^c \cap T)} \\ &= \frac{P(T|D)P(D)}{P(T|D)P(D) + P(T|D^c)P(D^c)} \\ &= \frac{(.8)(.4)}{(.8)(.4) + (.2)(.6)} = \frac{8}{11} \end{aligned}$$

(b) This is unknown without some knowledge of the probability of winning given drug use.

(c) Inverting the expression from (a), we get

$$
\begin{aligned}
P(T|D^c) &= \frac{1}{P(D^c)}\left[\frac{P(T|D)P(D)}{P(D|T)} - P(T|D)P(D)\right] \\
&= \frac{1}{.6}\left[\frac{(1)(.4)}{.95} - (1)(.4)\right] = \frac{2}{57}
\end{aligned}
$$

5. Assume the three players make simultaneous decisions in successive rounds. Assume further a player only washes if she is *certain* her face is dirty. For simplicity, use binary notation to indicate the states; for example, denote the state where A and C have dirty faces as 101.

   (a) In state 100, Ann sees two clean faces, so knows hers is dirty. In state 001, Chloe washes in the first round, so A knows her face is clean. In other states (101,010,110,011,111) noone washes in the first round.

   In state 101, Ann can see that Bernice's face is clean, so hers is dirty. In the other states, neither Ann nor Chloe can tell, so neither washes. In subsequent rounds, no further information is revealed.

   Thus, Ann can tell if her face is dirty only in states 100, 001, and 101.

   (b) If everyone could see, then in each of 100, 010, 001, someone washes. Thus, if noone washes in the first round, then Ann and Chloe deduce (incorrectly) that at least two people have dirty faces. If either sees a clean face, she deduces that her's is dirty. If noone washes in the second round, then all deduce the state is 111.

   Thus, in all states but 010, A correctly deduces whether her face is dirty. In that state, she and Chloe wash unnecessarily in the second round.