A Theory of the Interday Variations in Volume, Variance, and Trading Costs in Securities Markets

F. Douglas Foster
S. Viswanathan
Duke University

In an adverse selection model of a securities market with one informed trader and several liquidity traders, we study the implications of the assumption that the informed trader has more information on Monday than on other days. We examine the interday variations in volume, variance, and adverse selection costs, and find that on Monday the trading costs and the variance of price changes are highest, and the volume is lower than on Tuesday. These effects are stronger for firms with better public reporting and for firms with more discretionary liquidity trading.

In a securities market in which there are differentially informed traders, market makers must cover their losses from transactions with informed traders by charging a spread from all traders. Bagehot (1971)

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was the first to suggest that the bid–ask spread could be explained by this adverse selection faced by the marker maker, and formal models of this phenomenon have been constructed by Glosten and Milgrom (1985) and Kyle (1985). The practical importance of adverse selection has been confirmed by the empirical studies of Glosten and Harris (1988), Foster and Viswanathan (1990), and Stoll (1989), who attributes 43 percent of the quoted bid–ask spread to adverse selection.

In this article, we analyze the implications of adverse selection in securities markets for the intertemporal behavior of trading volume, trading costs, and price volatility, when there is periodic variation in the information advantage of an informed trader. We argue that, because the price is an important source of information for uninformed liquidity traders, the informed trader has the greatest advantage when the market first opens; and, the longer the market is closed, the more significant is the advantage of the informed trader at the opening.[1] In particular, the weekend closing of the market causes the information advantage to be greatest on Mondays.

Our model is based on that of Kyle (1985), which incorporates a single informed trader, a competitive market maker, and a group of liquidity traders. We introduce repeated trading periods (days) into the continuous-time version of his model, and allow new information about the security to arrive every day. In our model, the private-information advantage of the informed trader is reduced through time by public information and the market maker's inferences from changes in the order flow.

In Section 1 of the article, we consider an individual with monopolistic access to information and show that, without public revelation, it is optimal for the individual to trade in such a way that the market depth or market maker's price response to new orders is the same and that prices are equally informative each day. When private information is to be publicly revealed at a later date, the informed individual must transact more intensely, causing the private information to be released more quickly. If, as we shall assume, private information is received throughout the week, while public information is received only on weekdays, the market maker's sensitivity to changes in the order flow decreases through the week. As a result, the variance of price changes also declines through the week. These results depend crucially on a lack of additional liquidity traders on Monday to offset the extra information from the weekend. These interday variations in variances and the market maker's actions, and their dependence on the quality of the public information, are the key results of this article.

[1] The price effects of market closing are examined in Oldfield and Rogalski (1980).

In Section 2, we consider the effect of giving some liquidity traders discretion to delay their trades for one day without cost, so that they can avoid trading when the adverse selection problem is most severe. We show that without public information, the informed trader acts so as to keep constant the trading costs of these discretionary liquidity traders (the costs are proportional to the market depth or market maker's price response to new orders), rendering their delay tactics futile. However, when some information is revealed publicly, the informed trader's optimal strategy no longer equalizes the costs of trading for the liquidity traders through the week. While the equilibrium trading pattern depends on the number of discretionary liquidity traders and the quality of public information, we show that there are always higher trading costs and lower volume on Monday relative to Tuesday. Because we allow the discretionary liquidity traders to postpone their trades by only one day, we find that with precise public information, the equilibrium trading pattern has two days of concentrated trading each week; with poor public information there is only one day of concentrated trading each week. Admati and Pfleiderer (1988) consider a model with many informed traders, in which all private information becomes public at the end of the trading session; they find, in contrast to our model, a single period of trade concentration.

We examine some alternate specifications of the model in Section 3 of the article, and present our conclusions in Section 4. All proofs are in the Appendix.

1. The Basic Model

1.1 Model description

Consider a single asset that trades in a continuous auction market that is open five days a week and is closed on weekends and at night. The day of the week is denoted by d, where $d = 1$ is a Monday, so that the market is open on days 1 through 5. On each trading day, the market is open from time $t = 0$ to time $t = 1$.

The first market participant is the market maker who observes the order flow and sets prices. The second is an informed trader who has private information about the asset's value.[2] Finally, there are many liquidity traders who face liquidity shocks and trade at the market price when the shocks occur. All traders are assumed to be risk neutral.

[2] The assumption of a single informed trader is quite strong. However, variations with many informed traders (in a continuous auction or a discrete version with between-day shocks) are very complicated.

Information about the asset's payoff enters the model from a public source or from a private signal observed by the informed trader. Public information is available in two ways: first, at the close of trading each day there is a noisy signal of the asset's payoff; and, second, every calendar quarter the asset's payoff is announced at the end of the day's trading. Immediately after the quarterly announcement, the payoff is distributed to the investors as a dividend. We refer to the joint event of the quarterly announcement and dividend as the quarterly report. We assume that the unconditional expected value of the asset's payoff is zero.

Each day (before $t = 0$ on trading days) the informed trader receives a signal, v_d, of the asset's payoff next quarter. The expected value of the asset's payoff given all past information, including the private signal, is v_d. At the time of the quarterly report, there is no information asymmetry and the expected payoff for the next quarter is zero. Hence, v_d is also the liquidation value of the asset if the informed trader's knowledge were made public. v_d is given by

$$v_d = v_{d-1} + \epsilon_d, \tag{1}$$

where ϵ_d is independent and normally distributed with mean zero and variance σ_ϵ^2.[3]

At the end of each trading day there is a noisy public signal of the value, z_d:

$$z_d = v_d + \gamma_d, \tag{2}$$

where γ_d is the noise that distinguishes the public signal from the informed trader's valuation. γ_d is an independent, normally distributed shock with mean zero and variance σ_γ^2. The daily signal is distinct from the quarterly report.

Let $x_{d,t}$ denote the holdings of the asset by the informed trader. Then the instantaneous market order (purchases or sales) is denoted by $dx_{d,t}$. We assume that just prior to the quarterly report, the informed trader places an order so that she does not hold any of the asset when the quarterly report is announced.[4]

Liquidity traders enter the market continuously during the day and transact at the prevailing market price. Define their holdings of the asset by $l_{d,t}$. Then the instantaneous net purchases or sales are denoted by

$$dl_{d,t} = \sigma_l \, dw, \tag{3}$$

[3] This is the reduced form of a model in which the payoff next quarter is the sum of the daily shocks observed by the informed trader: v_{d-1} is the sum of past daily shocks, and ϵ_d is the current shock.

[4] At the instant the quarterly report is released, there is no information asymmetry and the informed trader is indifferent about closing her position.

where dw is a Wiener process and σ_l is the instantaneous order submission rate of liquidity traders.

Finally, there is a market maker who observes the total instantaneous order flow ($dy_{d,t} = dx_{d,t} + dl_{d,t}$) and sets a price ($p_{d,t}$) so as to make zero expected economic rents. The market maker sees the sum of the orders each instant, not the individual components. Thus, the price is equal to the conditional expectation of the asset value given the observed order flow

$$p_{d,t} = E[v_d \mid dy_{d,t}, \Omega_{d,t^-}], \tag{4}$$

where Ω_{d,t^-} represents the market information just before the instant t.

1.2. Equilibrium

Following Kyle (1985), we solve for the Nash equilibrium in the game between the market maker and the informed trader, assuming linear strategies for the informed trader and the market maker.[5] In this section, we present the equations of change for the model, derive the equilibrium trading for each day (Theorem 1), and show how the trading will evolve between days (Lemma 1 and Theorem 2).

Suppose that the informed trader picks an order submission strategy of the form

$$dx_{d,t} = \beta_{d,t}(v_d - p_{d,t})\ dt, \tag{5}$$

where $\beta_{d,t}$ is the informed trader's intensity of trade. The informed trader's order submission is an increasing function of the difference between his assessment of the expected payoff and the current price.

Given the linear strategy of the informed trader, the market maker observes the total order flow and adjusts prices. Market efficiency (the zero profit condition) and the normality of the relevant variables imply

$$dp_{d,t} = \lambda_{d,t}\ dy_{d,t} = \lambda_{d,t}(dx_{d,t} + dl_{d,t}), \tag{6}$$

where $\lambda_{d,t}$ is the sensitivity of the price to the order flow. $1/\lambda_{d,t}$ is a measure of market depth, because larger values of $\lambda_{d,t}$ cause larger price adjustments for a given transaction size. The market maker learns from the net orders ($dy_{d,t}$), so if the informed trader places large orders, the market maker learns about v_d efficiently.

Given the linear strategies of expressions (5) and (6), $\Pi_{d,t}$, the cumulative profit to the informed trader, follows the process

[5] Using a limiting argument, Kyle (1985) shows that if prices are assumed to be linear, then the optimal strategies of the informed trader will be linear and vice versa.

$$
\begin{aligned}
d\Pi_{d,t} &= (v_d - p_{d,t})\ dx_{d,t} \\
&= \beta_{d,t}(v_d - p_{d,t})^2\ dt.
\end{aligned} \tag{7}
$$

The informed trader's expected profits for the day can be written as

$$
E[\Pi_d | \Omega_{d,0^-}] = E\left[\int_0^1 \beta_{d,t}(v_d - p_{d,t})^2\ dt \mid \Omega_{d,0^-}\right] = \int_0^1 \beta_{d,t}\Sigma_{d,t}\ dt, \tag{8}
$$

where $\Sigma_{d,t}$ is the variance of the full information liquidation value (v_d) around the price ($p_{d,t}$) on day d at time t. We denote $\Sigma_{d,0}$ by Σ_d and $\Sigma_{d,1}$ by Λ_d. Σ_d is a measure of the stock of information that the informed trader brings to the market at opening, and Λ_d is a measure of the residual information kept from the market by the informed trader at the close of trading. The difference $\Sigma_d - \Lambda_d$ is a measure of the information released by the informed trader through her transactions on day d. The informed trader's stock of information changes from day to day according to

$$
\Sigma_{d+1} = \frac{\sigma_\gamma^2}{\sigma_\gamma^2 + \Lambda_d}\Lambda_d + \sigma_\epsilon^2. \tag{9}
$$

Equation (9) shows that the stock of information taken into day $d + 1$ consists of the private shock seen by the informed trader plus any private information from the close of trading on day d, adjusted for the information revealed by the public signal in day d.[6]

With this structure, we solve for the informed trader's optimal intraday trading strategy and present it as Theorem 1.

Theorem 1. *Given the opening variance (Σ_d) and the closing variance (Λ_d),*

$$
\lambda_{d,t} = \lambda_d = \left(\frac{\Sigma_d - \Lambda_d}{\sigma_l^2}\right)^{1/2},
$$

$$
\beta_{d,t} = \frac{(\sigma_l^2(\Sigma_d - \Lambda_d))^{1/2}}{\Lambda_d + (\Sigma_d - \Lambda_d)(1 - t)},
$$

$$
\Sigma_{d,t} = \Lambda_d + (\Sigma_d - \Lambda_d)(1 - t),
$$

$$
\Pi_d = \frac{\lambda_d}{2}\sigma_l^2 + \frac{1}{2\lambda_d}(\Sigma_d - \Lambda_d).
$$

[6] Kyle (1986) studies the effect of the public signal on the informed trader's strategy in an intraday model where public information arrives continuously through the day.

The sensitivity of the price to the order flow (λ_d) is constant through the day.

Theorem 1 shows that the sensitivity of the price to the order flow (λ_d) increases with the amount of information released by the informed trader ($\Sigma_d - \Lambda_d$) and falls with the amount of liquidity trading (σ_l^2). From the expression for $\beta_{d,t}$, we know that the informed trader transacts more intensely when there are more liquidity transactions to disguise her trades. Also, the expression for $\Sigma_{d,t}$ implies that the informed trader releases her information smoothly through the day. Finally, the informed trader's profit (Π_d) increases with the amount of liquidity trading and the amount of private information released through trading each day.

Armed with this result, we consider how the informed trader transacts on different days of the week. More vigorous trade releases more information on that day, which lowers subsequent trading profits, while the public information release causes the private information to depreciate between days. As a result, the informed trader will not carry information forward unless it can be used more effectively in future trading. By computing the trading profits on different days, we characterize the informed trader's actions and the market depth during the week. Lemma 1 shows that the informed trader will avoid days when the market maker is more sensitive to changes in the order flow. Theorem 2 uses Lemma 1 to derive the interday differences in the amount of information released through trading and market depth (allowing for the existence of the quarterly report on some weeks). In the remainder of this section, we discuss Lemma 1, then introduce Theorem 2 and prove the theorem by construction. Finally, we derive the interday patterns in market depth and price informativeness when the daily public signal is noninformative ($\sigma_\gamma^2 = \infty$).

To determine her optimal trading strategy, and hence the amount of information that will be released through trading, the informed trader compares her profit across days. Her total profit for day d and day $d + 1$ is

$$\Pi_d + \Pi_{d+1} = \frac{\lambda_d}{2}\sigma_l^2 + \frac{1}{2\lambda_d}(\Sigma_d - \Lambda_d) + \frac{\lambda_{d+1}}{2}\sigma_l^2 + \frac{1}{2\lambda_{d+1}}(\Sigma_{d+1} - \Lambda_{d+1}). \qquad (10)$$

Given the initial information stock (Σ_d), and the amount of information kept at the end of the next day's trading (Λ_{d+1}), the informed trader chooses the amount of information to be carried forward to tomorrow (Λ_d). Using (9), the first-order condition for an interior maximum of (10) is

$$-\frac{1}{\lambda_d} + \frac{1}{\lambda_{d+1}} \frac{\sigma_\gamma^4}{(\sigma_\gamma^2 + \Lambda_d)^2} = 0. \tag{11}$$

The second-order condition is

$$-2\frac{1}{\lambda_{d+1}} \frac{\sigma_\gamma^4}{(\sigma_\gamma^2 + \Lambda_d)^3} < 0. \tag{12}$$

We use expression (11) to solve for the informed trader's optimal trading strategy for the week.

Lemma 1. *For days d and $d+1$, with no quarterly report, if $\lambda_d < \lambda_{d+1}$, then $\Lambda_d = 0$. If $\lambda_d > \lambda_{d+1}$ and $\sigma_\gamma^2 \neq 0$, then $\Lambda_d > 0$.*

Lemma 1 states that if the market maker is more sensitive to changes in the order flow tomorrow, then the informed trader reveals all of her information through trading today. If the market maker is less sensitive to changes in the order flow tomorrow, then the informed trader reserves some of her information for trading tomorrow. Exceptions occur when public information is a perfect substitute for the informed trader's information.

Intuitively, one might expect the market maker to be more sensitive to changes in the order flow on Monday than on Friday, because the additional information shocks from the weekend give the informed trader a greater advantage on Monday. If the market maker is more sensitive to changes in the order flow on Monday, then Lemma 1 tells us that the informed trader will not carry information from Friday to Monday. It is this pattern induced by the weekend break that allows us to determine interday patterns in market depth and the informativeness of prices.

Theorem 2. *When $0 < \sigma_\gamma^2 < \infty$, and for weeks without a quarterly report, the unique equilibrium has*

$$\Sigma_1 - \Lambda_1 > \Sigma_2 - \Lambda_2 > \Sigma_3 - \Lambda_3 > \Sigma_4 - \Lambda_4 > \Sigma_5 - \Lambda_5 > \sigma_\epsilon^2,$$

$$\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4 > \lambda_5,$$

$$\Sigma_1 = 3\sigma_\epsilon^2,$$

$$\Lambda_5 = 0.$$

For weeks with a quarterly report on day d the unique equilibrium has

$$\Sigma_1 - \Lambda_1 > \cdots > \Sigma_d - \Lambda_d > \Sigma_{d+1} - \Lambda_{d+1} = \Sigma_5 - \Lambda_5 = \sigma_\epsilon^2,$$
$$\lambda_1 > \cdots > \lambda_d > \lambda_{d+1} = \lambda_5,$$
$$\Sigma_1 = 3\sigma_\epsilon^2,$$
$$\Lambda_d = \Lambda_{d+1} = \Lambda_5 = 0.$$

Theorem 2 says that in the absence of a quarterly report, the amount of information released through trading and the market maker's price sensitivity parameter (λ_d) decline monotonically over the week. For weeks with a quarterly report, more information is released through trading prior to the report. In this case, prices are more sensitive to the order flow prior to the quarterly report, and prices are less sensitive to the order flow after the quarterly report.

Assume that all information is exhausted by Friday close every week ($\Lambda_5 = 0$). Next, suppose that $\Lambda_d = 0$ for all days. Then Theorem 1 implies that $\lambda_1 = (3\sigma_\epsilon^2/\sigma_l^2)^{1/2}$ and $\lambda_2 = (\sigma_\epsilon^2/\sigma_l^2)^{1/2}$. However, by Lemma 1 this implies $\Lambda_1 > 0$, which is a contradiction. Thus, it pays the informed trader to carry some of her information from Monday to Tuesday, because the market is deeper and yields higher expected trading profits on Tuesday. A similar argument holds for other days of the week.

Now consider the informed trader's decision on Thursday. We know that some information is carried forward to Thursday ($\Sigma_4 > \sigma_\epsilon^2$) and that the first-order condition is

$$-\frac{1}{\lambda_4} + \frac{1}{\lambda_5} \frac{\sigma_\gamma^4}{(\sigma_\gamma^2 + \Lambda_4)^2} = 0. \tag{13}$$

Substitute the expressions for λ_4 amd λ_5 from Theorem 1 into expression (13):

$$-\left(\frac{\sigma_l^2}{\Sigma_4 - \Lambda_4}\right)^{½} + \left(\frac{\sigma_l^2}{\Sigma_5}\right)^{½} \frac{\sigma_\gamma^4}{(\Lambda_4 + \sigma_\gamma^2)^2} = 0. \tag{14}$$

If the information is exhausted on Thursday ($\Lambda_4 = 0$), then the left-hand side of expression (14) is positive. If no trades are made by the informed trader on Thursday ($\Lambda_4 = \Sigma_4$), then the left-hand side of equation (14) is infinitely negative. Because the left-hand side of expression (14) is decreasing in information withheld on Thursday (Λ_4), a unique solution exists. Also, as more information is carried into Thursday (Σ_4 increases), only a portion of it is withheld from trading (both Λ_4 and $\Sigma_4 - \Lambda_4$ increase). Finally, more information is released through trading on Thursday than on Friday, so the price is more sensitive to the order flow on Thursday (i.e., $\Sigma_4 - \Lambda_4 > \Sigma_5$, which implies $\lambda_4 > \lambda_5$).

Continue the construction for the remaining days of the week, using the first-order condition from expression (11). By induction, we find

$$\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4 > \lambda_5. \tag{15}$$

Because $\lambda_5 < \lambda_1$, the assumption $\Lambda_5 = 0$ is verified and Theorem 2 is proved.

Finally, it is interesting to consider the case of a useless daily public signal ($\sigma_\gamma^2 = \infty$). In weeks without a quarterly report, the first-order condition for the informed trader is

$$-\frac{1}{\lambda_d} + \frac{1}{\lambda_{d+1}} = 0. \tag{16}$$

The optimal solution has the same price sensitivity to the order flow each day,

$$\lambda_d = \left(\frac{7\sigma_\epsilon^2}{5\sigma_l^2}\right)^{½}, \tag{17}$$

and the informed trader transacts so that she releases the same amount of information each day ($\Sigma_d - \Lambda_d$ is the same every day).

In weeks with a quarterly report on day *d*, the informed trader exhausts her information through trading on day *d* because it will be useless after the quarterly report, so $\Lambda_d = 0$. For the remainder of the week there is the same amount of new information available each day, so the informed trader will equate the trading cost and release the same amount of information through trading on the remaining days ($\lambda_{d+1} = \cdots = \lambda_5$ and $\Sigma_{d+1} - \Lambda_{d+1} = \Sigma_5 - \Lambda_5 = \sigma_\epsilon^2$).

1.3. Variance implications

In order to analyze the patterns in the variance of price changes, define the interday change in closing prices from day d to day $d + 1$ as

$$r_{d,d+1} = p_{d+1,1} - p_{d,1}. \tag{18}$$

It is understood that if $d = 5$, then $d + 1 = 1$, the next Monday. The intraday variance of price changes for day $d + 1$ is [using expression (9)]

$$\Sigma_{d+1} - \Lambda_{d+1} = \sigma_\epsilon^2 + \frac{\sigma_\gamma^2}{\sigma_\gamma^2 + \Lambda_d}\Lambda_d - \Lambda_{d+1}. \tag{19}$$

Equation (19) states that the information released through trading each day is given by the total stock of information at the beginning of the day (the new private signal plus the information from the prior day after the public signal has been observed) less the stock of information kept at the close of trading. From this expression, the interday variance of price change from day d to $d + 1$ is

$$\mathrm{Var}(r_{d,d+1}) = \frac{\Lambda_d}{\sigma_\gamma^2 + \Lambda_d} \Lambda_d + \Sigma_{d+1} - \Lambda_{d+1} = \sigma_\epsilon^2 + \Lambda_d - \Lambda_{d+1}. \quad (20)$$

The right-hand side of Equation (20) is a measure of the information released from day d through day $d + 1$ by trading on day $d + 1$ and from the public signal after the close on day d. It shows that the information released through trading is given by the private signal plus the change in the stock of information held at the close of the day. The interday patterns in the variance of the changes in closing prices are listed in Theorem 3.

Theorem 3. *If the public signal is imperfectly informative ($0 < \sigma_\gamma^2 < \infty$), and there is no quarterly report during the week, then*

$$\mathrm{Var}(r_{5,1}) > \mathrm{Var}(r_{1,2}) > \mathrm{Var}(r_{2,3}) > \mathrm{Var}(r_{3,4}) > \mathrm{Var}(r_{4,5})$$

and

$$\mathrm{Var}(r_{5,1}) < 3\,\mathrm{Var}(r_{d,d+1}), \qquad \text{for } d = 1, 2, 3, 4.$$

If the public signal is uninformative ($\sigma_\gamma^2 = \infty$), then

$$\mathrm{Var}(r_{5,1}) = \mathrm{Var}(r_{d,d+1}), \qquad \text{for } d = 1, 2, 3, 4.$$

Theorem 3 states that when the public signal is useless, the variance of price changes is the same on all days. The reason for this is that if the information does not depreciate overnight, it pays for the informed trader to carry the information forward until the cost of exploiting the information is the same across days. Remember that the trading costs are related to the sensitivity of price to order flow (λ_d), and that, from Theorem 1, λ_d is a constant, and therefore the information revealed through trading each day ($\Sigma_d - \Lambda_d$) is constant. However, with an informative public signal, the informed trader's information depreciates each day. Thus, the informed trader will carry forward her information only if price sensitivity to order flow is lower the next day ($\lambda_{d+1} < \lambda_d$). From Theorem 1, this implies that the variance of price changes falls through the week, and Monday has a larger variance of price changes than other days. Interday differences in the variance of price changes can be interpreted as a statement about the strength of the public reporting in the market.

French and Roll (1986) observe that the variance of returns from Friday close to Monday close is less than three times the variance of returns from Monday close to Tuesday close. The ratio of the two-day weekend to weekday returns variance is 1.107 (see their table 1).

Table 1
A list of numerical solutions for the French and Roll (1986) ratio and the proportion of total information not released on Monday (Λ_1/Σ_1) without discretionary liquidity trading

σ_γ^2	French–Roll ratio	Λ_1/Σ_1
0.05	2.9393	0.0052
0.10	2.8830	0.0102
0.50	2.5455	0.0452
1.00	2.2812	0.0804
2.00	1.9728	0.1338
3.00	1.7924	0.1738
4.00	1.6717	0.2055
5.00	1.5843	0.2314
6.00	1.5179	0.2532
7.00	1.4654	0.2719
8.00	1.4227	0.2881
9.00	1.3874	0.3023
10.00	1.3576	0.3149
15.00	1.2584	0.3612
20.00	1.2024	0.3911

Parameter values of $\sigma_\epsilon^2 = 1.00$ (daily information shock) and $\sigma_\zeta^2 = 1.00$ (variance from liquidity traders' orders) are used. The variance of the public information signal observed each trading day is denoted by σ_γ^2.

They note that such a small ratio may result from the actions of privately informed traders. Our model is a rigorous version of their private-information hypothesis, and its predictions are consistent with their evidence.[7]

French and Roll (1986) use proportional returns to compute their ratios, whereas our model simply uses price changes. Foster and Viswanathan (1990) compute the French and Roll (1986) ratio using the variance of common stock price changes on the New York Stock Exchange in 1986. They show that while the ratio (in price changes or returns) can be sensitive to the time period considered, there are no significant differences in the estimates from the use of returns versus price changes.

In order to determine the parameter values that are needed to yield results that are consistent with the variance ratio observed by French and Roll (1986), Table 1 provides numerical solutions to the model for various values of the relative noise of the daily public signal ($\sigma_\gamma^2/\sigma_\epsilon^2$).[8] The ratio $\sigma_\gamma^2/\sigma_\epsilon^2$ needs to be 20.00 to generate a French and Roll (1986) ratio of 1.2024. At this value, the informed trader carries 39 percent of her information over from Monday to Tuesday. For small

[7] French and Roll (1986, p. 10) realize the importance of this carry-forward effect for the private information hypothesis: "... the private information hypothesis says that the variance pattern we observe occurs ... because informed investors usually trade when the exchange is open and they trade on their information for more than one day."

[8] The ratio $\sigma_\gamma^2/\sigma_\epsilon^2$ measures the noise to signal in the model and is the only relevant free parameter in the model [this can be shown from Equations (9) and (11)].

firms, this may be a plausible value; for larger firms, it may be less reasonable.

Finally, when the public information is less precise, the variance of price changes on Monday decreases and the variance of price changes increases on days later in the week.

Theorem 4. *Suppose $\sigma^2_{\gamma a} > \sigma^2_{\gamma b}$, that is, the precision of the public information increases from one environment to another. For the equilibria corresponding to these two parameter values,*

$$\Sigma_{1_a} - \Lambda_{1_a} \leq \Sigma_{1_b} - \Lambda_{1_b},$$

$$\Sigma_{5_a} \geq \Sigma_{5_b},$$

with a strict inequality in weeks without a quarterly report. Also, the informed trader has higher profits with $\sigma^2_{\gamma a}$.

2. The Model with Discretionary Liquidity Traders

In this section, we examine the implications of allowing discretionary liquidity trading.[9] The analysis is organized as follows. First, we compute expected net trading costs for the discretionary liquidity traders. Then, we determine a method for selecting the equilibrium trading pattern of the discretionary liquidity traders. This allows us to relate the actions of the discretionary liquidity traders to the quality of the daily public signal. We find that there are two days of concentrated trading when the public information is accurate (Theorem 5), and one day of concentrated trading when the public information is poor (Theorem 6). In Theorem 7 we show that discretionary liquidity traders will not trade on Monday if there is an informative daily public signal. Theorem 8 shows that the actions of the discretionary liquidity traders will determine the interday differences in trading volume. In addition, we provide numerical solutions to the model and examine the French and Roll (1986) ratio when there is discretionary liquidity trading.

2.1. Model description

In Section 1, liquidity traders did not recognize the informed trader's actions and simply traded when they experienced a liquidity shock. In practice, however, we expect that some liquidity traders will avoid days when the market maker is more sensitive to changes in the order flow. Admati and Pfleiderer (1988) show that discretionary liquidity

[9] Admati and Pfleiderer (1988) also consider the implications of discretionary liquidity trading, but in their model private information cannot be carried over from day to day.

traders move to a single period, resulting in one period of concentrated trading.

We allow discretionary liquidity traders to postpone their trades without cost by one calendar day, so that trades may be postponed within the week, but not over the weekend. Each discretionary liquidity trader is assumed to enter the market once a week, and must choose immediately whether to trade at the current time or trade the next day at the same time.[10] For simplicity, we assume that all discretionary liquidity traders follow the same rule each day: either all shift or all stay. To further simplify our analysis, we assume that the quarterly report is made at the close of trading on a Friday.

Denote discretionary liquidity traders by the subscript c, and nondiscretionary liquidity traders by the subscript n. At each instant of time there is a nondiscretionary liquidity trader with an order of $\sigma_n \, dn_{d,t}$ and a discretionary liquidity trader with an order of $\sigma_c \, dc_{d,t}$. We assume that instantaneous-order submission rates are independent.

If a discretionary liquidity trader whose instantaneous order is $\sigma_c \, dc_{d,t}$ trades on day d, the net expected trading cost is

$$\begin{aligned} -E[(v_d - p_{d,t} - dp_{d,t})\sigma_c \, dc_{d,t} \mid \Omega_{d,t^-}, dc_{d,t}] \\ = E[dP_{d,t}\, \sigma_c \, dc_{d,t} \mid \Omega_{d,t^-}, dc_{d,t}] = \lambda_d \sigma_c^2 (dc_{d,t})^2. \end{aligned} \tag{21}$$

Similarly, the discretionary liquidity trader's cost of trading at day $d + 1$ is $\lambda_{d+1}\sigma_c^2(dc_{d,t})^2$. Because the discretionary liquidity trader's costs are proportional to λ_d, he compares λ_d and λ_{d+1} and trades on the day when the market maker is less sensitive to changes in the order flow.[11]

Costs to discretionary liquidity traders depend on the instantaneous variance of discretionary liquidity orders on day d, which we denote as $\sigma_{d,c}^2$ ($\sigma_{d,c}^2 = 2\sigma_c^2$, $\sigma_{d,c}^2 = \sigma_c^2$, or $\sigma_{d,c}^2 = 0$). The value of $\sigma_{d,c}^2$ depends on whether discretionary liquidity traders shift their trades from day $d - 1$ to day d, and whether discretionary liquidity traders on day d stay or move to day $d + 1$. We represent the instantaneous variance of all liquidity orders on day d as σ_d^2. This value could be $\sigma_d^2 = 2\sigma_c^2 + \sigma_n^2$, $\sigma_d^2 = \sigma_c^2 + \sigma_n^2$, or $\sigma_d^2 = \sigma_n^2$, depending on the trading choices of discretionary liquidity traders on days d and $d - 1$.

[10] The assumption of a maximum of one-calendar-day delay is ad hoc. Because we are not determining the reason for trade from primitive considerations, any assumption we make is tenuous. The basic intuition that follows does not depend on the number of days of delay allowed.

[11] This particular formulation assumes the independence of all liquidity trades. Admati and Pfleiderer (1988) analyze the implications of relaxing this assumption.

Table 2
Allowable trading patterns for discretionary liquidity traders

Reference no.	Monday	Tuesday	Wednesday	Thursday	Friday	Equilibrium status
1	1	1	1	1	1	I
2	1	1	1	0	2	I
3	1	1	0	2	1	I
4	1	1	0	1	2	I
5	1	0	2	1	1	D
6	1	0	1	2	1	D
7	1	0	1	1	2	D
8	1	0	2	0	2	D
9	0	2	1	1	1	D
10	0	1	2	1	1	D
11	0	1	1	2	1	E
12	0	1	1	1	2	E
13	0	2	1	0	2	E
14	0	1	2	0	2	E
15	0	2	0	2	1	E
16	0	2	0	1	2	E

Entries in the table are the relative instantaneous order submission rates of discretionary liquidity trading, where the value on an average day is unity. The column titled "Equilibrium status" lists whether the trading pattern is infeasible for a Nash equilibrium (I), dominated by another equilibrium trading pattern (D), or can be an equilibrium trading pattern (E). This determination is derived in Theorem 7.

2.2 Equilibria

In our model, a Nash equilibrium is a set of trading rules that is optimal for each trader when they take the others' best rule as given.[12] We focus on Nash equilibria that are symmetric among discretionary liquidity traders. Because discretionary liquidity traders can trade immediately or postpone their transaction, they have two choices on each of four days, so there are 2^4, or 16, possible trading patterns (these are listed in Table 2). In this model, there may be more than one symmetric Nash equilibrium, so we develop a method for choosing among the possible Nash equilibria trading patterns.

Before we discuss equilibrium trading by discretionary liquidity traders in detail, there are some straightforward results worth noting. First, if the daily public information is informative ($\sigma_\gamma^2 \neq \infty$), a constant level of orders from discretionary liquidity traders through the week is not a Nash equilibrium, because trading costs would decrease through the week, creating an incentive to delay trades. Second, if the public signal is useless ($\sigma_\gamma^2 = \infty$), then discretionary liquidity trading is futile. As in Section 1, the informed trader transacts so that information is released in proportion to liquidity orders submitted, which equalizes trading costs through the week. Thus, the decay of

[12] More specifically, a Nash equilibrium provides a rule by which discretionary liquidity traders decide whether to delay their trade each day, a selection by the informed trader of how much information she will reveal through trading each day, and a price adjustment rule for the market maker.

information through public reporting is a necessary condition for discretionary liquidity traders to be able to reduce their trading costs below those of nondiscretionary liquidity traders.

If there is more than one Nash equilibrium, we select the equilibrium trading pattern that is efficient among discretionary liquidity traders. If we define $\mathbf{W}$ as the set of equilibrium trading patterns, then an efficient equilibrium solves

$$\min_{w \in \mathbf{W}} \sum_{d=1}^{5} \lambda_d \sigma_{d,c}^2. \tag{22}$$

An efficient equilibrium for a given set of model parameters must first eliminate trading patterns that cannot support a Nash equilibrium. Such patterns require discretionary liquidity traders to shift their trades in a manner that is inconsistent with the actions of the informed trader. From the remaining trading patterns, we choose the efficient trading patterns that are Nash equilibria and have the lowest costs for discretionary liquidity traders for the week, but these equilibria may not be unique.

To describe the efficient equilibrium trading patterns, we characterize the efficient equilibria for precise public information in Theorem 5, and the equilibrium for noisy (but useful) public information in Theorem 6. In Theorem 7, we show that efficient equilibrium trading patterns have no discretionary liquidity trading on Monday.

Theorems 5 and 6 show that the precision of the public information is a critical determinant of the equilibrium trading pattern. For accurate public information and the ability to delay trades by one day, the discretionary liquidity traders will find it optimal to cluster their trades on two separate days of the week, and they will not trade on Monday. That is, they move away from Monday, yet because the public information is very valuable, they pool their trades before the end of the week. In fact, with the ability to delay their transactions by one day, they find it optimal to pool their trades on two days of the week. For poor public information, the discretionary liquidity traders will not trade on Monday, and they only pool their trades on Friday.

Theorem 5. *There exists a bound, δ, such that for all $\sigma_\gamma^2 < \delta$, any efficient equilibrium has*

(i) $\sigma_1^2 = \sigma_n^2$; and

(ii) at least two days out of Tuesday, Wednesday, Thursday, and Friday have $\sigma_d^2 = \sigma_n^2 + 2\sigma_c^2$.

Theorem 5 shows that, when the informed trader's information and public information are close substitutes, there are two days with concentrated trading each week, because the informed trader is less able to shift trades and it pays the liquidity traders to group together.

Under the conditions of Theorem 6, the informed trader is relatively free to alter her trading pattern, so a concentration of discretionary liquidity traders only occurs on Friday.

Theorem 6. *There exists a bound v such that for all $\sigma_\gamma^2 > v$, the unique Nash equilibrium has*

$$\sigma_1^2 = \sigma_n^2,$$

$$\sigma_d^2 = \sigma_n^2 + \sigma_c^2, \qquad \text{for } d = 2, 3, 4,$$

$$\sigma_5^2 = \sigma_n^2 + 2\sigma_c^2.$$

With noisy public information, the informed trader has greater incentives to carry her information forward, requiring $\lambda_d > \lambda_{d+1}$, for $d = 1$, 2, 3, 4, and yielding concentrated trading on Friday and no discretionary liquidity trading on Monday.

Theorem 7 demonstrates that when the daily public information is useful, an efficient equilibrium trading pattern has Monday discretionary liquidity traders moving to Tuesday. Intuitively, this shift occurs because there is more private information on Monday, making trading on Monday more expensive.

Theorem 7. *Of the 16 trading patterns listed in Table 2, 10 are not equilibrium trading patterns or are dominated trading patterns. The six remaining trading patterns have Monday liquidity orders lower than Tuesday liquidity orders. Four of these trading patterns are double peaked (i.e., they have two days with concentrated discretionary liquidity trading), whereas two are single peaked. The Monday trading costs are the highest of the week for these six remaining trading patterns.*

While Theorem 7 does not identify a particular equilibrium trading pattern, efficient trading patterns are those without discretionary liquidity trading on Monday.

2.3. Volume implications

In this section, we define trading volume and investigate how it changes with the model's parameters. In contrast to Admati and Pfleiderer (1988), we show that volume need not be high when the variance of price changes is high.

We use Admati and Pfleiderer's (1988) definition of trading volume. Because we need to measure all trades submitted by the various groups, the volume is

$$\text{Total volume} = \min\{S^+_{d,t}, S^-_{d,t}\}, \tag{23}$$

where

$$\begin{aligned} S^+_{d,t} &= \sum_{j\in\{i,l\}} s^+_{j,d,t}, \qquad S^-_{d,t} = \sum_{j\in\{i,l\}} s^-_{j,d,t}, \\ s^+_{j,d,t} &= \max\{s_{j,d,t}, 0\}, \qquad s^-_{j,d,t} = \max\{-s_{j,d,t}, 0\}, \end{aligned} \tag{24}$$

and $s_{j,d,t}$ is the order submitted by the jth group of traders at time t on day d. For liquidity traders $s_{l,d,t} = \sigma_d \, dw_{d,t}$, and for the informed trader $s_{i,d,t} = dx_{d,t}$.

The total volume consists of half the absolute value of orders from the informed and liquidity traders plus the orders not crossed between the informed and liquidity traders that are taken by the market maker:

$$\text{Total volume at day } d, \text{ time } t = \frac{1}{2} \sum_{j\in\{i,l\}} |s_{j,d,t}| + \frac{1}{2} \left| \sum_{j\in\{i,l\}} s_{j,d,t} \right| . \tag{25}$$

In this continuous-time model, liquidity orders arrive at each instant according to a Wiener process. Thus, the expected volume from the liquidity traders over any time interval is infinite and liquidity-order submission swamps informed-order submission. The informed trader's volume, while small, is driven by the discretionary traders' orders; when the discretionary liquidity traders submit more orders, the informed trader submits more orders. To avoid the problem of infinite expected trading volume for an interval of time, we scale the volume at each instant by a constant, the expected nondiscretionary liquidity volume at each instant of time, $(2/\sqrt{2\pi})\sigma_n\sqrt{dt}$. Theorem 8 lists the interday volume differences with this scaling.

Theorem 8. *For any of the equilibrium trading patterns of Theorem 7, if $\sigma^2_d < (>) \, \sigma^2_{d+1}$, then*

Expected informed volume on day d, time t
  $<$ $(>)$ Expected informed volume on day $d + 1$, time t;

and

Expected total scaled volume on day d, time t
  $<$ $(>)$ Expected total scaled volume on day $d + 1$, time t.

It follows that for any of the equilibrium trading patterns of Theorem 7, the scaled volume is the lowest on Monday. When the public information is very precise (low σ^2_γ), the informed trader trades so that the volatility of price changes on Monday is high. This is the opposite of Admati and Pfleiderer's (1988) model: they suggest that trading volume and the variance of price changes move together.

2.4. Numerical solutions

To illustrate the implications of the model, we solve it numerically for a variety of parameter values and compare the results to those of Table 1. The parameters of the model are σ_ϵ^2, σ_γ^2, σ_c^2, and σ_n^2; only the relative noise of the daily public signal $(\sigma_\gamma^2/\sigma_\epsilon^2)$ and the portion of liquidity traders that is discretionary (σ_c^2/σ_n^2) are important in computing the equilibrium trading costs, volumes, and variances (they determine $\Lambda_d/\sigma_\epsilon^2$). We fix σ_ϵ^2 at unity and vary the remaining two parameters—σ_γ^2 from 0 to 6 in unit intervals and σ_c^2/σ_n^2 from 0 to 1 in intervals of 0.2—while keeping the total amount of liquidity trading constant at unity (i.e., $\sigma_c^2 + \sigma_n^2 = 1.00$).[13]

We calculate efficient equilibria for each of the various parameter values. With poor public information and few discretionary traders, trading pattern 12 of Table 2 (0, 1, 1, 1, 2) is the efficient equilibrium.[14] With precise information and many discretionary traders, trading pattern 14 of Table 2, (0, 1, 2, 0, 2), is chosen.

Figure 1 presents the French and Roll (1986) ratio as a function of the strength of the public signal $(\sigma_\gamma^2/\sigma_\epsilon^2)$ and the relative number of discretionary liquidity traders (σ_c^2/σ_n^2). For low $\sigma_\gamma^2/\sigma_\epsilon^2$ the French and Roll (1986) ratio is high, because public information is a good substitute for private information. The most interesting aspect of Figure 1 is the dramatic reduction in the French and Roll (1986) ratio when discretionary liquidity traders are included. For example, when $\sigma_c^2/\sigma_n^2 = 1.00$ and $\sigma_\gamma^2/\sigma_\epsilon^2 = 4.00$, the ratio drops to 1.1375. In Table 1 (without discretionary liquidity traders) the ratio did not fall to this level even with $\sigma_\gamma^2/\sigma_\epsilon^2 = 20.00$.

3. Alternative Specifications

In this section, we report numerical solutions of other specifications of the model. We alter the distribution of liquidity traders over trading days and reduce the amount of information accumulated over the weekend. For each of these trading scenarios, the basic results of Section 2 are unchanged.

[13] As both parameter ratios become close to zero, the first-order condition behaves poorly. Hence, for all of our calculations, we start at 0.0001 (close to zero) and use the increments described above.

[14] The (0, 1, 1, 1, 2) notation is a representation of discretionary liquidity orders on each day of the week. A 0 indicates no discretionary liquidity trading; a 2 indicates that discretionary liquidity traders from the prior day have shifted their trades ahead, and that the current discretionary liquidity traders are not postponing their trades.

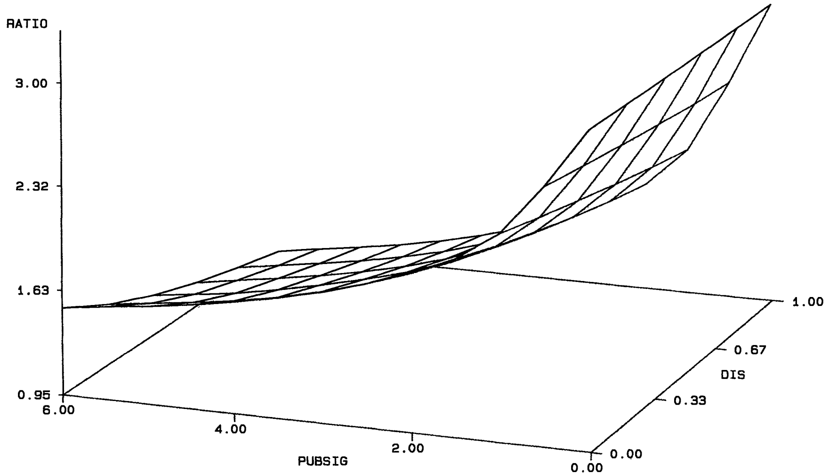

Figure 1
French and Roll (1986) Ratio
The plot illustrates that the French and Roll (1986) ratio (RATIO is the variance of Friday close to Monday close price changes divided by the variance of the Monday close to Tuesday close price changes) is sensitive to the quality of the daily public information (PUBSIG is $\sigma_\gamma^2/\sigma_\epsilon^2$) and the amount of discretionary liquidity trading (DIS is σ_c^2/σ_n^2). A large number of discretionary liquidity traders and a poor daily public signal dramatically lowers the French and Roll (1986) ratio.

3.1. Initial distribution of liquidity traders

Interday variations in volume, costs, and variance depend, in part, on the arrival rate of liquidity traders. In this section, we consider equal arrival rates of liquidity traders and private information. Because we have assumed that the private information continues to arrive on the weekend and that there is no trading or public information made available at this time, there is an accumulation of three days of private information at the Monday open. Here, we assume that the liquidity needs of the uninformed traders follow the same schedule on the weekend as during the week, which results in liquidity orders on Monday at three times the rate of other days of the week ($\sigma_1^2 = 3\sigma_2^2$). Without discretionary liquidity trading, the informed trader will release three times the information through trading on Monday; hence, the cost of trading is constant through the week, and the French and Roll (1986) ratio is 3.0, irrespective of the quality of the public information.

If we allow discretionary trading by a portion of the liquidity traders, introduction of three times as many liquidity traders on Monday as on other days has a significant effect on the French and Roll (1986) ratio. We solve the model numerically for the same range of values of $\sigma_\gamma^2/\sigma_\epsilon^2$ and σ_c^2/σ_n^2 used in Section 2.4. With poor public information ($\sigma_\gamma^2/\sigma_\epsilon^2 = 6.00$) and a large number of discretionary liquidity traders

($\sigma_c^2/\sigma_n^2 = 1.00$), the French and Roll (1986) ratio is much lower with the additional liquidity from the weekend (i.e., 0.4708, whereas in Section 2.4 it is 0.9492).

3.2. Private information accumulation over the weekend

Private information need not accumulate over the weekend at three times the weekday rate. Apart from agricultural commodities for which the weather plays a role, there seems to be no reason to exclude other accumulation rates. For the key results of our model, more information must accumulate over the weekend than on a weekday.

We repeat the calculations of Section 2.4 with a weekend information shock that is twice as large as the weekday shock. With a perfect daily public signal and no discretionary liquidity traders, $\sigma_\gamma^2/\sigma_\epsilon^2 = 0.00$ and $\sigma_c^2/\sigma_n^2 = 0.00$, the French and Roll (1986) ratio is 2.00. Reducing the quality of the public information, so that $\sigma_\gamma^2/\sigma_\epsilon^2 = 6.00$ and $\sigma_c^2/\sigma_n^2 = 0.00$, gives a French and Roll (1986) ratio of 1.2761. Finally, an increase in the number of discretionary liquidity traders to $\sigma_\gamma^2/\sigma_\epsilon^2 = 6.00$ and $\sigma_c^2/\sigma_n^2 = 1.00$ gives a ratio of 0.7953.

By lowering the amount of information released over the weekend, the French and Roll (1986) ratio drops. However, the extent of the drop (the relative informativeness of Monday prices) is still strongly affected by the number of discretionary liquidity traders.

4. Summary

In this article, a model of weekly trading patterns in which differentially informed traders behave strategically is presented. First, we exclude discretionary trading by liquidity traders, assume that the informed trader receives information each weekday, and find that the informed trader has a greater advantage on Mondays. We also show that the informed trader's profits are affected by the quality of the daily public signal. Without public information, the informed trader carries information from Monday to other days, so that the sensitivity of the price to the order flow is the same every day. When there is an informative daily public signal, which causes the private information to depreciate overnight, the informed trader does not carry as much information across days; hence, more information is released through trading early in the week, and prices are less sensitive to changes in the order flow later in the week.

Second, we allow some liquidity traders to delay their trade without cost for one day, and vary the quality of the public information. In the absence of public information, we find that delay tactics by discretionary liquidity traders are futile, because the informed trader acts to ensure that trading costs are constant across days. With a highly informative public signal, the presence of discretionary liquidity traders yields an efficient equilibrium with two days of concentrated trading each week. With less valuable public information, Friday is the only day with concentrated trading.

The implications of the model are that trading costs are highest on Monday and trading volume is low on Monday.[15] Because the strength of these effects depends on the quality of the public information, we expect that the changes in the interday volume, trading costs, and the variance of price changes will be strongest for actively traded, high-profile firms. In addition, because more discretionary liquidity trading accentuates the interday variations, the volume effects should be more pronounced for block trades (since block traders are more likely to be sophisticated in their trading).[16]

Appendix

Proof of Theorem 1

Given Σ_d, Λ_d, and the market maker's trading sensitivity $\lambda_{d,t}$, the informed trader maximizes

$$\max_{\beta_{d,t}} E\left[\int_0^1 d\Pi_{d,t} \mid \Omega_{d,0-}\right]$$

$$\text{s.t. } \Sigma'_{d,t} = -2\lambda_{d,t}\beta_{d,t}\Sigma_{d,t} + (\lambda_{d,t})^2\sigma_l^2, \tag{A1}$$

where $\Sigma'_{d,t}$ is the time derivative of $\Sigma_{d,t}$. The constraint comes from evaluating $\Sigma_{d,t+dt}$ and $\Sigma_{d,t}$, taking their difference, and letting $dt \to 0$. We simplify the objective function to

$$\begin{aligned} E\left[\int_0^1 d\Pi_{d,t} \mid \Omega_{d,0-}\right] &= E\left[\int_0^1 \beta_{d,t}(v_d - p_{d,t})^2 \; dt \mid \Omega_{d,0-}\right] \\ &= \int_0^1 E[\beta_{d,t}(v_d - p_{d,t})^2 \mid \Omega_{d,0-}] \; dt \\ &= \int_0^1 \beta_{d,t}\Sigma_{d,t} \; dt. \end{aligned} \tag{A2}$$

[15] Jain and Joh (1988) and Foster and Viswanathan (1990) show that for NYSE stocks, the Monday volume is lower than other days of the week. Also, Barclay, Warner, and Litzenberger (1988) show that for Japanese stocks, daily volume is low if the market had recently been closed (they consider the effects of Saturday trading on the Tokyo exchange). They note that their evidence on volume and variance is consistent with the private-information hypothesis.

[16] Both of these implications are examined and confirmed in Foster and Viswanathan (1990).

Similarly, given the informed trader's trading rule $\beta_{d,t}$, the market maker sets $\lambda_{d,t} = \beta_{d,t}\Sigma_{d,t}/\sigma_l^2$.

We show that only a constant $\lambda_{d,t}$ is possible in a Nash equilibrium.[17] To do this, we first argue (by contradiction) that $\lambda_{d,t}$ must be strictly positive for a Nash equilibrium. This implies that $\beta_{d,t}$ is strictly positive. Next, we prove that these two conditions are sufficient to ensure that $\Sigma'_{d,t} > 0$ for a Nash equilibrium. Then we show that among all Lebesgue measurable functions, only a constant $\lambda_{d,t}$ can occur in a Nash equilibrium. Having done this, we solve for $\beta_{d,t}$, which completes the proof.

If $\lambda_{d,t}$ is negative or zero over a set of positive measure on [0, 1], then the informed trader sets $\beta_{d,t}$ to be infinite in this region, and receives infinite trading profits. However, the market efficiency condition requires that a positive $\beta_{d,t}$ occurs with a positive $\lambda_{d,t}$, which is a contradiction. Hence, $\lambda_{d,t}$ must be strictly positive in a Nash equilibrium.

Next, any $\lambda_{d,t}$ consistent with a Nash equilibrium must ensure that the optimal strategy for the informed trader has a strictly positive $\beta_{d,t}$ and a strictly negative $\Sigma'_{d,t}$. The strictly positive $\beta_{d,t}$ follows from $\lambda_{d,t} > 0$ and the market efficiency condition. We prove that $\Sigma'_{d,t} < 0$ by contradiction. Suppose that $\Sigma'_{d,t} \geq 0$. Using the constraint in expression (A1) and knowing that $\beta_{d,t} > 0$ results in

$$0 < \beta_{d,t} \leq \frac{\lambda_{d,t}\sigma_l^2}{2\Sigma_{d,t}}. \tag{A3}$$

From the market efficiency condition we know that the last term in (A3) is $\beta_{d,t}/2$. This substitution into expression (A3) yields $1 \leq \frac{1}{2}$, which is a contradiction.

Because the informed trader determines $\beta_{d,t}$, Equation (A1) implies that she determines $\Sigma'_{d,t}$. Hence, we can view $\Sigma'_{d,t}$ as the control variable and substitute for $\beta_{d,t}\Sigma_{d,t}$ from the constraint of (A1) into the objective function

$$\max_{\Sigma d,t} \int_0^1 -\left(\frac{\Sigma'_{d,t}}{2\lambda_{d,t}} - \frac{\lambda_{d,t}\sigma_l^2}{2}\right) dt. \tag{A4}$$

To show that $\lambda_{d,t}$ is constant, suppose that there is a Nash equilibrium with two time intervals in the day, A and B (of measure $\mu A > 0$ and $\mu B > 0$, respectively), where $\lambda_{d,t} > \lambda > \lambda_{d,t'}$ for all $t \in A$ and $t' \in B$. We know that a Nash equilibrium must have $\Sigma'_{d,t} < 0$ for all t. We can

[17] We are grateful to Kent Daniels for suggesting this argument.

make the informed trader better off (relative to the Nash equilibrium) as follows. Set $\hat{\Sigma}'_{d,t} = \Sigma'_{d,t}$ for $t \notin A$ and $t \notin B$. For $t \in A$, let $\hat{\Sigma}'_{d,t} = 0$. For $t \in$ B,

$$\hat{\Sigma}'_{d,t} = \Sigma'_{d,t} + \frac{1}{\mu B} \int_{t \in A} \Sigma'_{d,t}\, dt. \tag{A5}$$

This new strategy is feasible and

$$\begin{aligned}
\int_{t \in A} - \frac{\Sigma'_{d,t}}{2\lambda_{d,t}}\, dt &< \int_{t \in A} - \frac{\Sigma'_{d,t}}{2\lambda}\, dt \\
&= -\frac{1}{2\lambda} \int_{t \in A} \Sigma'_{d,t}\, dt \\
&= -\frac{\mu B}{2\lambda} \int_{t \in A} \frac{\Sigma'_{d,t}}{\mu B}\, dt \\
&= \int_{t' \in B} -\frac{1}{2\lambda} \left(\int_{t \in A} \frac{\Sigma'_{d,t}}{\mu B}\, dt \right) dt' \\
&< \int_{t' \in B} - \frac{1}{2\lambda(t')} \left(\int_{t \in A} \frac{\Sigma'_{d,t}}{\mu B}\, dt \right) dt'.
\end{aligned} \tag{A6}$$

This series of inequalities uses the fact that in a Nash equilibrium the informed trader has at least one optimal strategy, where $\Sigma'_{d,t}$ is strictly negative. The inequalities imply that the informed trader has greater profits with the new strategy. Therefore, the optimal strategy must have $\Sigma'_{d,t} = 0$ and $\beta_{d,t} = 0$ over the set A. The market maker would then set $\lambda_{d,t} = 0$ in the set A, which contradicts our requirement that $\lambda_{d,t}$ be positive in A. Hence, the only $\lambda_{d,t}$ consistent with a Nash equilibrium is a constant, which we call λ_d.

With $\lambda_{d,t} = \lambda_d$, the informed trader's objective function becomes

$$\frac{1}{2\lambda_d} [\Sigma_d - \Lambda_d] + \frac{\lambda_d \sigma_l^2}{2}, \tag{A7}$$

and any trading intensity $(\beta_{d,t})$ where $\Sigma_{d,0} = \Sigma_d$ and $\Sigma_{d,1} = \Lambda_d$ is optimal. We use the market efficiency condition to choose the equilibrium trading intensity ($\lambda_d = \beta_{d,t}\Sigma_{d,t}/\sigma_l^2$ implies $\beta_{d,t}\Sigma_{d,t} = \lambda_d \sigma_l^2$) and substitute this into our expression for $\Sigma'_{d,t}$,

$$\Sigma'_{d,t} = -\lambda_d^2 \sigma_l^2, \tag{A8}$$

and then integrate:

$$\Sigma_{d,t} = -\lambda_d^2 \sigma_l^2 t + k. \tag{A9}$$

When $t = 0$, $\Sigma_{d,0} = \Sigma_d = k$. When $t = 1$,

$$\Lambda_d = \Sigma_{d,1} = -\lambda_d^2 \sigma_l^2 + \Sigma_d \tag{A10}$$

or

$$\lambda_d = \left(\frac{\Sigma_d - \Lambda_d}{\sigma_l^2}\right)^{1/2} \tag{A11}$$

By substitution we have

$$\Sigma_{d,t} = -(\Sigma_d - \Lambda_d)t + \Sigma_d = (\Sigma_d - \Lambda_d)(1 - t) + \Lambda_d. \tag{A12}$$

We use the market efficiency condition to obtain

$$\beta_{d,t} = \frac{\lambda_d \sigma_l^2}{\Sigma_{d,t}} = \frac{\{(\Sigma_d - \Lambda_d)\sigma_l^2\}^{1/2}}{(\Sigma_d - \Lambda_d)(1 - t) + \Lambda_d}. \tag{A13}$$

■

Proof of Lemma 1

If $\lambda_d < \lambda_{d+1}$, then at $\Lambda_d = 0$, the left-hand side of expression (11) is negative. From the concavity of the objective function in Λ_d, $\Lambda_d = 0$ is the optimal solution.

If $\lambda_d > \lambda_{d+1}$, $\sigma_\gamma^2 \neq 0$, then the first-order condition (11) is not satisfied at $\Lambda_d = 0$, so $\Lambda_d > 0$. If $\sigma_\gamma^2 = 0$, the public information reveals the informed trader's information completely and $\Lambda_\mathrm{d} = 0$. ■

Proof of Theorem 2

The proof of uniqueness uses an induction argument. Suppose two quarterly reports are in adjacent weeks. This corresponds to $n = 1$ of the induction argument. We label these as weeks 1 and 2. For this proof we use a pair of subscripts to denote the week and day. With this notation, $\Lambda_{1,5} \neq 0$ implies that $\lambda_{1,5} > \lambda_{2,1}$ and $\Sigma_{2,1} - \Lambda_{2,1} > \Sigma_{2,1}^* - \Lambda_{2,1}^* > \sigma_\epsilon^2$, where the * superscript corresponds to the equilibrium with $\Lambda_{1,5} = 0$. Suppose day d is the reporting date in week 1. If d is the last day of the week when $\Lambda_d = 0$ (if not it is easy to amend the proof), then $\Sigma_{d+1} - \Lambda_{d+1} > \Sigma_{1,5} - \Lambda_{1,5}$. This yields $\lambda_{1,5} < \lambda_{2,1}$, which is a contradiction.

Suppose the above is true for n weeks; that is, if the two reporting dates have n weekends between them, all these weekends satisfy $\lambda_{k,5} < \lambda_{k+1,1}$, $k = 1, \ldots, n$. Consider the case where the two reporting dates have $n + 1$ weekends separating them. Now $\Lambda_{1,5} > 0$ implies that $\lambda_{1,5} > \lambda_{2,1}$. Thus $\sigma_\epsilon^2 > \Sigma_{1,d+1} - \Lambda_{1,d+1} > \cdots > \Sigma_{1,5} - \Lambda_{1,5}$, where d is the reporting day in week 1. Either $\lambda_{2,1} > \cdots > \lambda_{n+1,1}$ or there is a (k, s) such that $\lambda_{k,s} < \lambda_{k,s+1}$ If the second, the induction hypothesis proves the theorem [$\Lambda_{k,s} = 0$, which is equivalent to reporting on day (k, s)]. If the first, we use the fact that $\Sigma_{2,1} - \Lambda_{2,1} > \cdots > \Sigma_{n+1,1} -$

$\Lambda_{n+1,1} > \sigma_\epsilon^2$ to contradict $\lambda_{1,5} > \lambda_{2,1}$. Hence $\lambda_{1,5} < \lambda_{2,1}$ and the induction hypothesis proves the same for the rest of the weekends ($\lambda_{k,5} < \lambda_{k+1,1}$, for $k = 2, \ldots, n$). ∎

Proof of Theorem 3

The facts that

$$\begin{aligned} &\Lambda_1 > \Lambda_2 > \Lambda_3 > \Lambda_4 > \Lambda_5, \\ &\Sigma_1 - \Lambda_1 > \Sigma_2 - \Lambda_2 > \Sigma_3 - \Lambda_3 > \Sigma_4 - \Lambda_4 > \Sigma_5 - \Lambda_5 > \sigma_\epsilon^2 \end{aligned} \tag{A14}$$

suffice to ensure

$$\mathrm{Var}(r_{1,2}) > \mathrm{Var}(r_{2,3}) > \mathrm{Var}(r_{3,4}) > \mathrm{Var}(r_{4,5}). \tag{A15}$$

To prove that $\Sigma_d - \Lambda_d > \sigma_\epsilon^2$, use an induction argument. Suppose $\Sigma_4 > \sigma_\epsilon^2$. If $\Lambda_4 = \Sigma_4 - \sigma_\epsilon^2$, then $\Sigma_5 > \sigma_\epsilon^2$ and $\lambda_4 < \lambda_5$; which is a contradiction. Therefore, $\Sigma_4 - \Lambda_4 > \sigma_\epsilon^2$. By induction assume that this is true for any $d + 1$. Then if $\Sigma_d > \sigma_\epsilon^2$, $\Lambda_d = \Sigma_d - \sigma_\epsilon^2$ implies that $\Sigma_{d+1} > \sigma_\epsilon^2$. By the induction hypothesis, $\Sigma_{d+1} - \Lambda_{d+1} > \sigma_\epsilon^2$. As a result, $\lambda_d < \lambda_{d+1}$, which is a contradiction.

To compare dates 1 and 2, use the first-order condition from expression (11):

$$\begin{aligned} \mathrm{Var}(r_{5,1}) &= \Sigma_1 - \Lambda_1 \\ &= \left(\frac{\Lambda_1 + \sigma_\gamma^2}{\sigma_\gamma^2}\right)^4 (\Sigma_2 - \Lambda_2) > \left(1 + 4\frac{\Lambda_1}{\sigma_\gamma^2}\right)(\Sigma_2 - \Lambda_2) \\ &= (\Sigma_2 - \Lambda_2) + \left(4\frac{\Lambda_1}{\sigma_\gamma^2}\right)(\Sigma_2 - \Lambda_2) \\ &> (\Sigma_2 - \Lambda_2) + \left(4\frac{\Lambda_1}{\sigma_\gamma^2}\right)\left(\frac{1}{4}\Sigma_2\right) \\ &> (\Sigma_2 - \Lambda_2) + 4\frac{\Lambda_1}{\sigma_\gamma^2}\left(\frac{\sigma_\gamma^2\Lambda_1}{4(\Lambda_1 + \sigma_\gamma^2)}\right) = \mathrm{Var}(r_{1,2}). \end{aligned} \tag{A16}$$

The bound on $(\Sigma_2 - \Lambda_2)$ is proved as follows. Let $\Sigma_2 = \sigma_\epsilon^2 + \delta$ and when $\sigma_\gamma^2 = \infty$, $\Lambda_2 = 3\delta/4$. When $\sigma_\gamma^2 < \infty$, $\Lambda_2 < 3\delta/4$. Thus, $\Sigma_2 - \Lambda_2 > \sigma_\epsilon^2 + \delta/4 > \Sigma_2/4$.

To obtain French and Roll's (1986) results, note that when $\sigma_\gamma^2 = 0$, $\mathrm{Var}(r_{5,1}) = 3\sigma_\epsilon^2$ and $\mathrm{Var}(r_{d,d+1}) = \sigma_\epsilon^2$, for $d \neq 5$. But if $\sigma_\gamma^2 > 0$, $\Sigma_1 - \Lambda_1 < 3\sigma_\epsilon^2$ and $\Sigma_{\mathrm{d}} - \Lambda_{\mathrm{d}} > \sigma_\epsilon^2$, for $d \neq 5$. As a result,

$$\mathrm{Var}(r_{5,1}) < 3\,\mathrm{Var}(r_{d,d+1}), \qquad \text{for } d = 1, 2, 3, 4. \tag{A17}$$

This completes the proof of the proposition for the case $\sigma_\gamma^2 \neq 0$. When $\sigma_\gamma^2 = \infty$, the proof of the theorem is obvious. ■

Proof of Theorem 4

We consider only weeks without a quarterly report. Suppose Σ_4 is fixed. If $\sigma_{\gamma a}^2 > \sigma_{\gamma b}^2$, the first-order condition implies that $\Lambda_{4_a} > \Lambda_{4_b}$. Therefore $\Sigma_4 - \Lambda_{4_a} < \Sigma_4 - \Lambda_{4_b}$. Also $\Lambda_{4_a} > \Lambda_{4_b}$ and $\sigma_{\gamma a}^2 > \sigma_{\gamma b}^2$ imply that $\Sigma_{5_a} > \Sigma_{5_b}$. Λ_{4_b} is also a feasible trading pattern for $\sigma_{\gamma a}^2$, and it is more profitable for the informed trader with $\sigma_{\gamma a}^2$ than with $\sigma_{\gamma b}^2$. Hence, the informed trader's profits are larger with $\sigma_{\gamma a}^2$.

Assume that the above is true for $d = 2$. Let Σ_1 be given. The first-order condition between dates 1 and 2 is

$$-\left(\frac{\sigma_l^2}{\Sigma_1 - \Lambda_1}\right)^{1/2} + \left(\frac{\sigma_l^2}{\Sigma_2 - \Lambda_2}\right)^{1/2} \frac{\sigma_\gamma^4}{(\Lambda_1 + \sigma_\gamma^2)^2} = 0. \tag{A18}$$

By induction, and given Σ_2, we have $\Sigma_{2_a} - \Lambda_{2_a} < \Sigma_{2_b} - \Lambda_{2_b}$ and $\Sigma_{5_a} > \Sigma_{5_b}$. From the first-order condition we know $\Lambda_{1_a} > \Lambda_{1_b}$, which implies $\Sigma_1 - \Lambda_{1_a} < \Sigma_1 - \Lambda_{1_b}$. Because $\Lambda_{1_a} > \Lambda_{1_b}$ and $\sigma_{\gamma a}^2 > \sigma_{\gamma b}^2$, we have $\Sigma_{2_a} > \Sigma_{2_b}$. Using the induction assumption and the fact that Σ_5 is monotone in Σ_2, $\Sigma_{5_a} > \Sigma_{5_b}$. The statement on profits is left to the reader. ■

Proof of Theorem 5

For $\sigma_\gamma^2 = 0$ a trading pattern with $\sigma_1^2 = \sigma_n^2 + \sigma_c^2$ has $\lambda_1 = (3\sigma_\epsilon^2/(\sigma_n^2 + \sigma_c^2))^{1/2}$. If $\sigma_2^2 = \sigma_n^2 + \sigma_c^2$, then $\lambda_2 = (\sigma_\epsilon^2/(\sigma_n^2 + \sigma_c^2))^{1/2}$, and $\lambda_1 > \lambda_2$, which is not a Nash equilibrium.

If $\sigma_2^2 = \sigma_n^2$, then $\lambda_2 = (\sigma_\epsilon^2/\sigma_n^2)^{1/2}$, which may be a Nash equilibrium. We shift Monday's discretionary liquidity orders to Tuesday and use a ^ to denote values with the shifted orders,

$$\hat{\lambda}_1 = \left(\frac{3\sigma_\epsilon^2}{\sigma_n^2}\right)^{1/2} > \hat{\lambda}_2 = \left(\frac{\sigma_\epsilon^2}{\sigma_n^2 + \sigma_c^2}\right)^{1/2}, \tag{A19}$$

which is consistent with an equilibrium. On Wednesday, either

$$\hat{\lambda}_3 = \left(\frac{\sigma_\epsilon^2}{\sigma_n^2 + \sigma_c^2}\right)^{1/2} \quad \text{or} \quad \hat{\lambda}_3 = \left(\frac{\sigma_\epsilon^2}{\sigma_n^2 + 2\sigma_c^2}\right)^{1/2}. \tag{A20}$$

Therefore $\hat{\lambda}_3 \leq \hat{\lambda}_2$, which is needed for Tuesday's discretionary liquidity traders to shift to Wednesday. The rest of the trading pattern is unaffected by the shift from Monday to Tuesday. For this new trading pattern, discretionary liquidity traders arriving on Monday are better off and other discretionary liquidity traders are better off or unaffected. Thus $\sigma_1^2 = \sigma_n^2 + \sigma_c^2$ is a dominated trading pattern, which completes the proof of the first claim when $\sigma_\gamma^2 = 0$. Since pooling discretionary liquidity traders reduces the cost from $\sigma_\epsilon^2/(\sigma_n^2 + \sigma_c^2)$ to $\sigma_\epsilon^2/(\sigma_n^2 + 2\sigma_c^2)$, the second part of the proposition is also true. By the continuity of the equilibrium costs λ_d in σ_γ^2 for any given distribution of liquidity orders, there is a bound δ such that the proposition holds for $\sigma_\gamma^2 < \delta$. ■

Proof of Theorem 6

We prove that discretionary liquidity traders shift from Monday (i.e., $\sigma_1^2 = \sigma_n^2$). The remainder of the proof uses similar arguments.

If $\sigma_1^2 = \sigma_n^2 + \sigma_c^2$, then $\sigma_2^2 = \sigma_n^2$. For this to be an equilibrium trading pattern,

$$\left(\frac{3\sigma_\epsilon^2}{\sigma_n^2 + \sigma_c^2}\right)^{1/2} = \lambda_1 < \lambda_2 = \left(\frac{\sigma_\epsilon^2 - \Lambda_2}{\sigma_n^2}\right)^{1/2}. \tag{A21}$$

Let λ_d be the first day after Tuesday where $\lambda_d < \lambda_{d+1}$. Because on Friday, $\lambda_5 < \lambda_1$ (proved below), this is well-defined. Then, repeated use of the first-order condition implies

$$\lambda_2 = \lambda_d\left(1 + \frac{\Lambda_{d-1}}{\sigma_\gamma^2}\right)^2 \cdots \left(1 + \frac{\Lambda_2}{\sigma_\gamma^2}\right)^2. \tag{A22}$$

Because $\Lambda_{d-1}, \ldots, \Lambda_2$ are bounded numbers, as $\sigma_\gamma^2 \to \infty$, for any $\delta > 0$, $\lambda_2 < \lambda_d + \delta$ eventually holds. Hence, for any $\delta > 0$, there is a large enough $\nu(\delta)$ such that

$$\lambda_2 < \left(\frac{\sigma_\epsilon^2}{\sigma_n^2 + \sigma_c^2}\right)^{1/2} + \delta, \tag{A23}$$

for all $\sigma_\gamma^2 > \nu(\delta)$. By choosing

$$\delta = (\sqrt{2} - 1)\left(\frac{\sigma_\epsilon^2}{\sigma_n^2 + \sigma_c^2}\right)^{1/2},$$

we contradict $\lambda_1 < \lambda_2$. As a result, the trading pattern cannot be a Nash equilibrium. We use similar arguments for the remaining days and let ν be the maximum of the $\nu(\delta)$ corresponding to each day.

To show that $\lambda_5 < \lambda_1$, for any trading pattern with $\sigma_1^2 = \sigma_n^2 + \sigma_c^2$, there is one day of the week such that $\lambda_d < \lambda_{d+1}$. If $\Lambda_5 = 0$, it is easy to show that $\lambda_5 < \lambda_1$. If $\Lambda_5 > 0$, then $\Sigma_1 - \Lambda_1$ increases and $\Sigma_5 - \Lambda_5$ decreases, so $\lambda_5 < \lambda_1$ and $\Lambda_5 = 0$, which is a contradiction. ■

Proof of Theorem 7

For trading patterns 1, 2, 3, and 4 of Table 2, Monday discretionary liquidity traders stay on Monday, and Tuesday discretionary liquidity traders stay on Tuesday. This implies

$$\left(\frac{3\sigma_\epsilon^2}{\sigma_n^2 + \sigma_c^2}\right)^{1/2} = \lambda_1 > \left(\frac{\sigma_\epsilon^2}{\sigma_n^2 + \sigma_c^2}\right)^{1/2} = \lambda_2, \tag{A24}$$

which is inconsistent with a Nash equilibrium.

There are four trading patterns where Wednesday discretionary liquidity traders stay on Wednesday, and Thursday discretionary liquidity traders stay on Thursday (trading patterns 1, 5, 9, and 10 in Table 2). We eliminated trading pattern 1 in the prior paragraph. Compare trading pattern 10 (0, 1, 2, 1, 1) with trading pattern 14 (0, 1, 2, 0, 2). Trading pattern 10 is dominated by trading pattern 14, because discretionary liquidity traders arriving on Thursday are better off, while no other discretionary trader is worse off. Trading patterns 5 and 9 are dominated by trading patterns 8 and 13, respectively.

The three remaining trading patterns, where Monday discretionary liquidity traders stay on Monday, are (1, 0, 2, 0, 2), (1, 0, 1, 2, 1), and (1, 0, 1, 1, 2). All of these trading patterns require $\lambda_2 > (\sqrt{3}\sigma_\epsilon)/(\sigma_c^2 + \sigma_n^2)^{1/2}$, which limits their viability as equilibria. First, we provide bounds on σ_γ^2 beyond which these trading patterns are not equilibria. For trading pattern (1, 0, 2, 0, 2) the first-order condition requires $\lambda_2 = \lambda_3(1 + \Lambda_2/\sigma_\gamma^2)^2$. If $\sigma_\gamma^2 \geq 6\sigma_\epsilon^2$, then, because $\Lambda_2 \leq \sigma_\epsilon^2$, we obtain

$$\begin{aligned}\lambda_2 &< \lambda_3\left(1 + \frac{\sigma_\epsilon^2}{6\sigma_\epsilon^2}\right)^2 < \frac{\sigma_\epsilon}{(\sigma_c^2 + \sigma_n^2)^{1/2}}\left(\frac{7}{6}\right)^2 \\ &= 1.3611\frac{\sigma_\epsilon}{(\sigma_c^2 + \sigma_n^2)^{1/2}} < \sqrt{3}\frac{\sigma_\epsilon}{(\sigma_c^2 + \sigma_n^2)^{1/2}},\end{aligned} \tag{A25}$$

so Monday's discretionary liquidity traders will want to postpone their trades. Hence, (1, 0, 2, 0, 2) is not an equilibrium for $\sigma_\gamma^2 > 6\sigma_\epsilon^2$. The similar bound for trading pattern (1, 0, 1, 2, 1) is $7\sigma_\epsilon^2$ and for trading pattern (1, 0, 1, 1, 2) the bound is $11\sigma_\epsilon^2$.

Second, we prove that these trading patterns, when they are equilibria, are dominated by trading patterns (0, 1, 2, 0, 2), (0, 1, 1, 2, 1), and (0, 1, 1, 1, 2). We show that trading pattern (1, 0, 2, 0, 2) is dominated by (0, 1, 2, 0, 2); the proof for the other two trading patterns is similar [we use ^ to denote the parameter values for trading pattern (0, 1, 2, 0, 2)]. When $\sigma_\gamma^2 < 6\sigma_\epsilon^2$, $\hat{\Sigma}_2 < 3\sigma_\epsilon^2$. Consequently, $\hat{\lambda}_2 < (\sqrt{3}\sigma_\epsilon)/(\sigma_c^2 + \sigma_n^2)^{1/2} = \lambda_1 < \lambda_2$, so $\hat{\lambda}_3 < \lambda_3$. It is easy to verify that when (1, 0, 2, 0, 2) is an equilibrium and $\hat{\lambda}_2 < (\sqrt{3}\sigma_\epsilon)/(\sigma_c^2 + \sigma_n^2)^{1/2}$, then (0, 1, 2, 0, 2) is an equilibrium. Hence, (1, 0, 2, 0, 2) is dominated up to $6\sigma_\gamma^2$.

We prove that Monday has the lowest trading costs for pattern (0, 1, 2, 0, 2). Because $\lambda_1 > \lambda_2 > \lambda_3$ and $\lambda_4 > \lambda_5$, it suffices to prove that $\lambda_1 > \lambda_4$. Assume the opposite. If $\lambda_1 < \lambda_4$, then $\Lambda_1 > \Lambda_4 + 2\sigma_\epsilon^2$ and $\lambda_5 > \lambda_2 > \lambda_3$ from the first-order conditions. In fact, the first-order condition yields

$$\frac{\lambda_5}{\lambda_2} > \left(\frac{\sigma_\gamma^2 + \Lambda_4 + 2\sigma_\epsilon^2}{\sigma_\gamma^2 + \Lambda_4}\right)^2. \tag{A26}$$

Using $\lambda_5 < \sigma_\epsilon/(\sigma_c^2 + \sigma_n^2)^{1/2}$ and simplifying yields

$$\lambda_2\left(\frac{\sigma_\gamma^2 + \Lambda_4 + 2\sigma_\epsilon^2}{\sigma_\gamma^2 + \Lambda_4}\right)^2 < \frac{\sigma_\epsilon}{(\sigma_c^2 + \sigma_n^2)^{1/2}}. \tag{A27}$$

Substituting for λ_2 yields

$$(\Sigma_2 - \Lambda_2)\left(\frac{\sigma_\gamma^2 + \Lambda_4 + 2\sigma_\epsilon^2}{\sigma_\gamma^2 + \Lambda_4}\right)^4 < \sigma_\epsilon^2. \tag{A28}$$

$\lambda_5 > \lambda_3$ implies $\Lambda_2 < \Lambda_4 < \sigma_\epsilon^2$. Using this yields

$$\Sigma_2 - \Lambda_2 > (2\sigma_\epsilon^2 + \Lambda_4)\frac{\sigma_\gamma^2}{\sigma_\gamma^2 + 2\sigma_\epsilon^2 + \Lambda_4}. \tag{A29}$$

Substituting this into Equation (A28) gives a contradiction. For the other five trading patterns, it is simple to prove that trading costs are highest on Monday. ∎

Proof of Theorem 8

The instantaneous informed trader volume has expectation equal to $(2/\sqrt{2\pi})\sigma_{i,d,t}$, where

$$\begin{aligned}
\sigma_{i,d,t} &= \sqrt{E[\{\beta_{d,t}(v_d - p_{d,t})\}^2]}\ dt \\
&= \sqrt{\beta_{d,t}^2\Sigma_{d,t}}\ dt \\
&= \beta_{d,t}(\Sigma_{d,t})^{1/2}\ dt \\
&= \frac{\lambda_d\sigma_d^2}{[(\Sigma_d - \Lambda_d)(1 - t) + \Lambda_d]^{1/2}}\ dt \\
&= \frac{[\Sigma_d - \Lambda_d]^{1/2}\sigma_d}{[(\Sigma_d - \Lambda_d)(1 - t) + \Lambda_d]^{1/2}}\ dt.
\end{aligned} \tag{A30}$$

If liquidity orders on day d are less than that on day $d + 1$, then $\Lambda_d > 0$. The first-order condition of discretionary liquidity traders yields

$$\frac{\Sigma_d - \Lambda_d}{\sigma_d^2} = \frac{\Sigma_{d+1} - \Lambda_{d+1}}{\sigma_{d+1}^2}\left(1 + \frac{\Lambda_d}{\sigma_\gamma^2}\right)^4$$

$$\Longleftrightarrow \Sigma_d - \frac{\sigma_d^2}{\sigma_{d+1}^2}\Sigma_{d+1}\left(1 + \frac{\Lambda_d}{\sigma_\gamma^2}\right)^4$$

$$= \Lambda_d - \frac{\sigma_d^2}{\sigma_{d+1}^2}\Lambda_{d+1}\left(1 + \frac{\Lambda_d}{\sigma_\gamma^2}\right)^4$$

$$\Longleftrightarrow \frac{\Sigma_d}{\Sigma_{d+1}} - \frac{\sigma_d^2}{\sigma_{d+1}^2}\left(1 + \frac{\Lambda_d}{\sigma_\gamma^2}\right)^4$$

$$= \frac{\Lambda_{d+1}}{\Sigma_{d+1}}\left(\frac{\Lambda_d}{\Lambda_{d+1}} - \frac{\sigma_d^2}{\sigma_{d+1}^2}\left(1 + \frac{\Lambda_d}{\sigma_\gamma^2}\right)^4\right). \tag{A31}$$

Because $\Lambda_{d+1}/\Sigma_{d+1} < 1, \Sigma_d/\Sigma_{d+1} < \Lambda_d/\Lambda_{d+1}$. With $\sigma_d^2 < \sigma_{d+1}^2$, Equation (A30) yields that the informed volume on day *d*, time *t* is lower than that on day $d + 1$, time $t + 1$.

To prove the converse, note that $\sigma_d^2 > \sigma_{d+1}^2$ means $\Lambda_d = 0$, so the informed volume on day *d* is invariant to changes in σ_γ^2. The informed volume on day $d + 1$ is lower than when $\sigma_\gamma^2 = 0$, because $\Lambda_{d+1} > 0$. At $\sigma_\gamma^2 = 0$, the informed volume on day *d* is higher, which completes the proof. ■

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