Multimarket Trading and Market Liquidity

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When a security trades at multiple locations simultaneously, an informed trader has several avenues in which to exploit his private information. The greater the proportion of liquidity trading by "large" traders who can split their trades across markets, the larger is the correlation between volume in different markets and the smaller is the informativeness of prices. We show that one of the markets emerges as the dominant location for trading in that security. When informed traders can use their information for more than one trading period, the timely release of price information by market makers at one location adversely affects the profits informed traders expect to make subsequently at other locations. Market makers, competing to offer the lowest cost of trading at their location, consequently deter informed trading by voluntarily making the price information public and by "cracking down" on insider trading.

An informed trader typically has more than one avenue to exploit his private information. Many securities are now cross-listed on exchanges in this country and abroad. In addition, there are the so-called "third and

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fourth markets" in some stocks. Also, there often exist active markets in derivative securities, such as futures and options. An investor with private information about a stock could trade, for example, on one or more exchanges on which the stock is listed, while simultaneously trading in off-the-exchange markets and options markets.

All this trading should be thought of as taking place in a single market *only* in the extreme case of complete and continuous information sharing across markets. In general, however, the liquidity characteristics of the markets would be different and, as Grossman (1990) also points out, there would exist short-term disparities in the trading prices for the same security at different locations. The presence of profit-maximizing informed traders and cost-minimizing liquidity traders does, however, impose certain equilibrium restrictions on the prices and on the liquidity characteristics across markets.

Following Kyle (1985) and Admati and Pfleiderer (1988), we assume that competitive market makers absorb the net order flow submitted by the informed and the liquidity traders. We analyze a situation in which a security trades in multiple markets simultaneously. We start by assuming that the information possessed by informed traders is short-lived, so that there is essentially no "learning" between markets before this information becomes public. "Small" liquidity traders are assumed to trade their entire requirement on a single market. "Large" liquidity traders (such as institutions) split their trades across markets to minimize costs. The informed traders' order sizes, in equilibrium, are perfectly correlated across markets, as are those of the large liquidity traders. Therefore, the correlation between total trading volume across any two markets increases in the proportion of liquidity trading accounted for by the large traders and is shown, in equilibrium, to be identical for any pair of markets. This provides an interesting implication since the participation by institutions in securities markets has been increasing over time and small investors' share of trading has been steadily declining.[1]

The presence of small liquidity traders implies that the informed traders' expected profits are larger if all markets are not perfectly unified informationally. This occurs because trades by small liquidity traders are uncorrelated, making the aggregate size of a "typical" trade smaller when there is a single market. It is thus more difficult for an informed trader to camouflage his trades. If these markets are informationally separate, this diversification effect is lost. This increases the informativeness of prices since informed traders now trade more aggressively. However, informed traders do not benefit from the frag-

[1] See *Economist* (1990) and *Wall Street Journal* (1986).

mentation of large liquidity traders across markets since their trades are perfectly correlated.

Small liquidity traders, in our model, do not split their trades across markets. But some of them may have discretion concerning the market in which they transact. Naturally, they will choose the market in which their expected trading costs are the smallest. We show that small liquidity traders with discretion will concentrate, in equilibrium, in the market that has the largest amount of trading by liquidity traders who are unable to move between markets. This market will attract, in turn, more trading by the informed as well as the large liquidity traders. If a financial instrument trades on several locations simultaneously, this "winner takes most" feature results in concentration of trading to the location that has the largest number of traders with no discretion to move between markets. This concentration of trading result is analogous to that in Admati and Pfleiderer (1988). They show that liquidity traders with discretion over trading *periods* will tend to concentrate their trading in the same period. We show that trading patterns are sensitive to the relative distribution across markets of traders who do not have the flexibility to choose the location of their trading.

There has been increasing awareness in recent years about the need to improve information transmission between various markets trading identical or similar securities. An outcome of this concern has been the *Securities Acts Amendments of 1975,* which mandates the SEC to move rapidly toward the implementation of a truly nationwide competitive securities market. The objectives are to achieve greater efficiency, enhance competition, and improve execution of orders. While there has been some progress toward the establishment of linkages such as the *Consolidated Tape* and the *Intermarket Trading System,* considerable effort is needed to achieve a unified securities market.[2]

The question that we address is whether there exist economic forces that may lead to transmission of price information between trading locations even in the absence of regulatory action. We consider a scenario in which the private information of the informed traders is long-lived (i.e., they have more than one round of trading in which to exploit their information). We show that a location in which market makers make the price information public is less attractive to informed traders. The reason is that timely release of price information negatively affects the profits informed traders expect to make subsequently in other market locations as well. Since it reduces the adverse selection, market makers competing to offer the lowest cost of trading at

[2] See Amihud and Schwartz (1985), Bloch and Schwartz (1979), and Cohen et al. (1986) for a discussion of these and other related issues.

their location may *voluntarily* make the price information public to deter informed trading at their location.

We then examine the incentives to detect and deter insider trading in the absence of regulatory action. We consider a scenario in which the insider's private information is long-lived and each market location releases price information after each round of trading. A market location in which market makers "crack down" on insider trading leads to less aggressive trading by insiders. This is since insiders' expected profits from trading in that location are reduced, though the extent to which the insiders' private information is revealed through trading stays the same. Market makers, competing to offer the lowest cost of trading at their location, may therefore have the incentive to run "clean markets."[3] Also, since cracking down discourages informed trading, it may attract sufficiently large numbers of small liquidity traders to concentrate on the market in which market makers crack down on insiders. Consequently, this market may even attract the largest proportion of large liquidity as well as informed trading. This is contrary to the speculation in the popular press that cracking down on insider trading at one location may drive trading away to other locations.[4]

1. The Model

We consider a security that trades on N markets.[5] All agents are assumed to be risk neutral. There are two classes of traders—informed traders and liquidity traders. There is one informed trader who trades based

[3] An article in the *Wall Street Journal* (1987b) reports: "The New York Stock Exchange announced a review of the security industry's regulatory system to see what changes may be needed to protect investors and 'maintain the integrity of the market.'" Another article in the *Wall Street Journal* (1988a) reports: "Firms have developed several elaborate computer procedures to help compliance officers detect instances of unusual or insider trading." [See also *Wall Street Journal* (1987a, 1987c).]

[4] An article in the April 24, 1987 issue of the *Wall Street Journal* (1987d) quotes former Commissioner Bevis Longstreth: "If we don't bend, we run the risk of forcing our investors offshore, where our securities laws provide no investor protection at all, and of diminishing the U.S. as the dominant market." An editorial in the October 25, 1988 issue of the *Wall Street Journal* (1988c) in response to a proposed insider trading bill [see *Wall Street Journal* (1988b)] argues: "When the bill comes to the President for a signature or veto, he might think a bit about American competitiveness and the like, and remember that the less regulated London and Tokyo stock markets are now but a quick phone call away." [See also *Wall Street Journal* (1987e).]

These claims voiced in the popular press may have some validity if there are significant errors in detecting the true "insiders," in which case the "cracking down" may have a chilling effect on liquidity traders as well.

[5] For analytic simplicity we have taken it to be the same security trading on all the markets. We suspect our results generalize to nonidentical securities that are affected by common factors such as futures and forward contracts. However, we do not directly address information revelation differences across different assets [e.g., equity versus options or index contracts versus individual securities; see Kumar and Seppi (1990), Subrahmanyam (1991)].

on his private information.[6] The liquidity traders engage in trading for exogenous reasons. Following Kyle (1985), we assume that there are risk-neutral market makers in each of the N markets who trade on their own account and absorb any order imbalances.

Sequence of events:

- Each period, market makers in each market announce price schedules.
- On observing the price schedules, all traders submit their orders.
- On observing the net order flow, market makers in each of the N markets absorb the net order flow according to the announced price schedules.

Strategies:

- *Market makers:* As in Kyle (1985), risk-neutral market makers at each location engage in a Bertrand competition in price schedules.
- *Informed trader:* The informed trader privately observes the innovation in the value of the security. Given the announced price schedules, he places his orders in each of the N markets so as to maximize expected trading profits.
- *Liquidity traders:* Given the announced price schedules, liquidity traders also submit their orders in each of the N markets so as to minimize expected trading costs.

The equilibrium notion in our model is different from that in Admati and Pfleiderer (1988). The concept of equilibrium in their model is Nash in the strategies of *all* the players—under their assumptions, the market makers' role is passive. However, the role of competition among market makers in our model is central to the nature of the questions we are addressing. The focus of our analysis is to examine the implications of actions market makers take in order to attract order flow. It is imperative, therefore, that traders be allowed to react to moves by competing market makers. The sequence of events in our model formalizes this since market makers, in announcing price schedules, will take into account the effect of the price schedules on traders' order submissions.

In each of the N markets, Bertrand competition among market makers ensures that in equilibrium their announced price schedules are such that the market makers earn zero expected profits for any net order flow that might occur. In equilibrium, the set of announced price schedules must be such that no market maker can expect to

[6] Our results generalize to the case in which there is more than one but an *exogenously* fixed number of informed traders. We do not analyze, but will briefly discuss later, the case in which entry by informed traders is *endogenous*.

make positive profits by unilaterally deviating from his equilibrium strategy. A market maker, in considering a deviation, knows that traders can respond to such a move. The notion that traders respond to announced price schedules captures the intuition that traders do shop around to obtain the lowest possible transactions costs. For instance, we do observe dealers posting bid–ask spreads.

After traders have submitted their orders, the information sets of market makers contain only the net order flows, since they have no means of distinguishing between different types of traders. We assume that *all* market makers in a given market location observe the net order flow *only* in that market—it is essentially this feature that distinguishes one market location from another in the model.

1.1 One period: No cross-market observability

We first consider the case in which the informed trader has only one round of trading in which to exploit his private information. The information becomes public knowledge before the next round of trading. Trading takes place simultaneously in each of the N markets.

In this section, we assume that there are two types of liquidity traders: small and large. Each small liquidity trader buys or sells small quantities of shares on the one market in which he trades. These are analogous to the discretionary liquidity traders in the basic model of Admati and Pfleiderer (1988). The notion here is that since they trade small quantities, the expected effect on security price is relatively small compared to the time and the fixed costs associated with trading in multiple markets. There is also a large (e.g., institutional) liquidity trader who takes into account the effect of his trades on the prices he pays; this trader splits his trades across markets to minimize the total expected cost of trading a given quantity.[7] The large liquidity trader is analogous to the discretionary liquidity traders in Admati and Pfleiderer (1988, section 6): in their model, these traders are able to allocate trades across different *periods.* We consider both types of liquidity traders simultaneously. Our approach in this section is similar to that in Bhushan (1991), which, in the context of a multi-*asset* model, also gives an optimizing role to the liquidity traders. In later sections, we abstract away from this distinction between small and large liquidity traders, since it is not central to the issues discussed there.

At any time t, let

$\tilde{v}^t \equiv$ innovation in the value of the security at time t,

[7] Our results are not affected by the assumption that there is only one large liquidity trader.

$\tilde{u}_i^t$ ≡ aggregate order flow by small liquidity traders in market i at time t,
$\tilde{d}^t$ ≡ order flow by the large liquidity trader across all N markets at t.

We assume that the distributions of $(\tilde{v}, \{\tilde{u}_i\}_{i=1}^N, \tilde{d})$ are identical each period (allowing us to suppress the superscript t) and are independent normal with

$$E\tilde{v} = 0, \qquad \mathrm{Var}\,\tilde{v} \equiv \sigma_v^2;$$

$$E\tilde{u}_i = 0, \qquad \mathrm{Var}\,\tilde{u}_i \equiv \sigma_i^2 \qquad \forall i;$$

$$E\tilde{d} = 0, \qquad \mathrm{Var}\,\tilde{d} \equiv \sigma_d^2.$$

We let

x_i ≡ size of the order in market i by the informed trader,

d_i ≡ size of the order in market i by the large liquidity trader,

$$\sum_{i=1}^{N} d_i \equiv d.$$

Let

P^0 ≡ unconditional value of the security before the current round of trading begins,
P_i ≡ price charged by the market makers in market i.

Since the market makers in market i observe the net order flow only in their own market and cannot distinguish between types of traders, their pricing function depends only on the net order flow, $x_i + u_i + d_i$. We initially assume that the market makers have linear pricing rules. We later show that the linear pricing rules are indeed consistent with equilibrium.

The market makers in market i, thus, use the following pricing rule:

$$\Delta P_i \equiv P_i - P^0 = \lambda_i(x_i + u_i + d_i), \tag{1}$$

where λ_i denotes the depth of the market i. Notice that, in general, the prices charged in different markets would be different.

1.1.1 Small liquidity traders without discretion. Given the pricing rules in (1), the large liquidity trader allocates the quantity he trades across different markets so as to minimize the total cost of trading. He solves the following problem:

$$\min_{\{d_i\}_{i=1}^N} \sum_{i=1}^{N} E\left[d_i \lambda_i(x_i^* + \tilde{u}_i + d_i)\right] \qquad \text{such that } \sum_{i=1}^{N} d_i = d,$$

where x_i^* denotes the optimal quality traded by the informed trader. Let $\{d_i^*\}_{i=1}^N$ denote the solution to the above problem. The first-order conditions for the minimization problem yield the following:

$$d_i^* = k_i d + \frac{1}{2}(k_i - 1)\sum_{j=1}^{N} Ex_j^*(v), \tag{2}$$

where

$$k_i \equiv \frac{c_i}{\Sigma_{i=1}^N c_i} \qquad \forall i, \tag{3}$$

$$c_i \equiv \frac{1}{2\lambda_i} \qquad \forall i. \tag{4}$$

Similarly, taking as given the pricing rules in (1), the informed trader solves the following maximization problem:[8]

$$\max_{\{x_i\}_{i=1}^N} \sum_{i=1}^{N} E\left[x_i\{v - \lambda_i(x_i + \tilde{u}_i + k_i\tilde{d})\}\right].$$

The first-order conditions for the maximum problem are

$$x_i^* = c_i v, \qquad \forall i.$$

Notice that the informed trader's strategies are linear in v. Since $Ex_i^*(v) = 0$, $\forall i$, substituting in (2) we get $d_i^* = k_i d$.

The market makers in the ith market set the change in price ΔP_i equal to the expectation of $\tilde{v}$ conditional on having observed the net order flow, $x_i^* + u_i + k_i d$. This ensures that the expected profits of the market makers, conditional on observing the net order flow, are equal to zero. Normality makes the regressions linear and we get

$$c_i\sigma_v^2 = \lambda_i(c_i^2\sigma_v^2 + \sigma_i^2 + k_i^2\sigma_d^2), \qquad \forall i. \tag{5}$$

The equilibrium, then, is defined by (3), (4), and (5).

Lemma 1.

$$\lambda_i = \frac{1}{2}\frac{(\Sigma_{i=1}^N \sigma_i)}{\sigma}\frac{\sigma_v}{\sigma_i} \qquad \forall i, \tag{6}$$

$$c_i = \frac{\sigma}{(\Sigma_{i=1}^N \sigma_i)}\frac{\sigma_i}{\sigma_v} \qquad \forall i, \tag{7}$$

[8] Our results generalize to a case when the informed trader (or an exogenously fixed number of informed traders) observes the signal v with some noise. The analysis with an *endogenous* number of informed traders whose signals are *not* perfectly correlated is quite complex and is not undertaken here.

$$k_i = \frac{\sigma_i}{(\sum_{i=1}^{N} \sigma_i)} \qquad \forall i, \tag{8}$$

where

$$\sigma \equiv \left[\sigma_d^2 + \left(\sum_{i=1}^{N} \sigma_i\right)^2\right]^{1/2}. \tag{9}$$

Proof. See the Appendix.

These results are similar to those in Bhushan (1991), where he develops a multiasset model that demonstrates an interdependence in trading costs arising from the optimizing behavior of liquidity traders.

The informed trader's orders across all markets are perfectly correlated. The same is also true of the large liquidity trader's orders. The correlation ρ between the *total* order flows on any two markets is given by the following.

Proposition 1.

$$\rho(x_i^* + u_i + k_i d, x_j^* + u_j + k_j d) = \frac{1}{2}\left[1 + \frac{\sigma_d^2}{\sigma^2}\right], \qquad \forall i \forall j,\ i \neq j.$$

Proof. See the Appendix.

Notice that the correlation is identical for any pair of markets. Also, the correlation is increasing in σ_d/σ, which measures the relative size of trades by the large liquidity trader to the size of the total trading by all liquidity traders. The intuition is that the trades by the informed as well as the large liquidity trader are perfectly correlated across markets, while the trades of the small liquidity traders are uncorrelated across markets. As the importance of liquidity trading by the large liquidity trader increases, so does the correlation. In the limit, when there is no trading by the small liquidity traders, $\sigma_d/\sigma = 1$ and the correlation approaches 1.

We now examine the informativeness of prices resulting from trading in the security. We consider a measure of informativeness of prices, ψ, which we define as the ratio of the variance of the innovation conditional on observing all prices P_i's to the unconditional variance of the innovation.

Proposition 2. *The informativeness of prices ψ is given by*

$$\psi \equiv \frac{\text{Var}\left[\tilde{v} \mid \{\Delta P_i\}_{i=1}^{N}\right]}{\sigma_v^2} = \frac{N}{(N + 1) + (\sigma_d^2/\sigma^2)(N - 1)}.$$

Proof. See the Appendix.

We can make a number of observations. First, notice that if the number of markets equals 1, then one half of the informed trader's private information is reflected in prices, as in Kyle (1985). Second, if there were no small liquidity traders (in the limit), then σ_d^2/σ_v^2 equals 1, and the measure of price informativeness equals $\frac{1}{2}$ regardless of the number of markets. In general, however, the informativeness of prices increases as the number of markets increase, and is higher when the proportion of liquidity trading accounted for by small liquidity traders is higher; for a given number of markets, the informativeness is the highest when the proportion of liquidity trading by the large trader is zero.

The intuition for these results is as follows. When there is only one market, since orders of small liquidity traders are uncorrelated, the size of a typical aggregate trade by the small liquidity trader is smaller than the sum of their typical aggregate trades across all markets when there is more than one market. As a result, the informed trader trades more aggressively when there are a larger number of markets—because when small liquidity traders are spread across a larger number of markets, some of the diversification effect resulting from aggregation is lost. As a result of aggressive trading by the informed trader, more of his private information gets incorporated in prices. No such diversification effect exists for the trading by the large liquidity trader since his trades across markets are perfectly correlated. So, for the limiting case of no trading by small traders, the informed trader does not benefit from an increase in the number of markets.

1.1.2 Some small liquidity traders with discretion. So far, the distribution of small liquidity traders across markets was assumed exogenous. We now consider the situation in which at least some of the small liquidity traders can choose the market in which they trade. We assume that small liquidity traders do not split their trades across markets, possibly because the order is too small and there are fixed transactions costs associated with trading in each market. However, these traders are able to trade in the one market in which their expected trading costs are smallest (i.e., the market with the smallest λ).

Let

$n_i \equiv$ number of small traders who must trade in market i (let us call them *noise* traders),

$m \equiv$ number of small traders who can choose the market in which they trade.

For simplicity, take the trades of each small trader to be independently normally distributed with mean zero and variance equal to unity. If we let $m_i \equiv$ number of small traders with discretion who trade in market i, then

$$\sigma_i = (n_i + m_i)^{1/2}.$$

If we let $\{m_i^*\}_{i=1}^{N}$ denote the distribution, in equilibrium, of small liquidity traders with discretion across all N markets, we obtain the following result.

Proposition 3. *Let market k denote a market with the largest number of noise traders. Then, an equilibrium exists which is characterized as follows. All small traders with discretion trade in market k. Formally,*

$$m_k^* = m, \qquad m_i^* = 0, \qquad \forall i \neq k.$$

Market makers in each of the N markets announce the values of λ's that are consistent with this equilibrium distribution of discretionary liquidity traders and with zero profits.

Generically, this equilibrium is unique.

Proof. First consider the case in which there is a unique market, k, that has the largest number of noise traders. This is the generic case. All small liquidity traders with discretion would prefer to trade in a market that offers the smallest λ. There are several possible candidates for equilibrium. Consider a situation in which market j offers the lowest λ across the N markets. In equilibrium all m traders will trade in market j. This scenario cannot be an equilibrium because, as we show below, market makers in the market with the largest number of noise traders (i.e., market k) can undercut this market by announcing an even lower value of λ. The announcement of this lower value of λ would attract all m traders away from market j to market k. Notice that we do not require small traders with discretion to coordinate their actions since they simply respond to the *announcements* of the λ's by market makers in different markets. We show that market makers in market k can indeed offer the lowest value of λ that is also consistent with zero expected profits.

First, notice that the expression for λ_i is a product of two terms, $\frac{1}{2}\sigma_v/\sigma_i$ and $(\Sigma_{i=1}^{N} \sigma_i)/\sigma$. All m traders concentrating in market k minimizes the first term. If there were no liquidity traders, the second term equals 1 and we obtain the concentration result that is analogous to the one in Admati and Pfleiderer (1988). We now show that all m

traders concentrating in market k also minimizes the second term over *all possible* allocations of m traders to N markets. To see this, notice that since σ_i is concave in $(n_i + m_i)$, $\Sigma_{i=1}^{N} \sigma_i$ is minimized if we allocate all m traders to market k since n_k is the largest of all n_i's. This also minimizes $(\Sigma_{i=1}^{N} \sigma_i)/\sigma$ since it is increasing in $\Sigma_{i=1}^{N} \sigma_i$.

This equilibrium is stable and is not broken by any other deviations since a market maker in no other market can announce a lower λ and still be able to make nonnegative profits.

Now consider the case when there is more than one market with the largest number of noise traders. Without loss of generality, let l denote another market that also has the largest number of noise traders. It is easy to verify that the equilibrium described in the statement of the proposition exists—since no market maker can unilaterally deviate and make positive expected profits. Similarly, another equilibrium with $m_l^* = m$ also exists. However, a scenario with market makers in both markets k and l announcing identical values of λ's cannot be an equilibrium. ■

So, in equilibrium, small discretionary traders will concentrate their trading to a single market:

$$\sigma_k = (n_k + m)^{1/2}, \qquad \sigma_i = (n_i)^{1/2}, \qquad \forall i \neq k.$$

Since σ_i is largest for the market k, and the trades of the large liquidity and of the informed trader in market i are proportional to σ_i (Lemma 1), this market also gets the largest share of trades by the informed and the large liquidity traders. In the limit, when the proportion of nondiscretionary noise traders in the pool of small liquidity traders becomes negligible, all trading is concentrated in one market. Notice, however, that as long as there is some noise trading in a market, the large liquidity trader does allocate some trading to this market.

We wish to point out the differences between this result and the ones in Admati and Pfleiderer (1988). In their basic model, they get the concentration of trading to one period when their liquidity traders are not allowed to split their trades across periods. When they allow the liquidity traders to split their trades (in section 6), they show that one may still obtain some concentration in trading, but the reason is that the order flows from earlier periods are informative in later periods. In our context, since trading is *simultaneous* across different markets, we will not get any concentration in trading if all our liquidity traders could split their trades across markets. We are thus pointing out the importance of liquidity traders who cannot split their trades for concentration in trading results to obtain. We consider both types of liquidity traders—those who split their trades across markets and those who do not—simultaneously and show that trading patterns are sensitive to the relative distribution of noise traders across markets and to the relative size of the small traders with discretion as compared to the size of the noise traders. Since our structure is simpler, in the sense that there is no cross-market information flow, we are also able to get a closed-form solution for our equilibrium.[9]

There seems to be a natural monopoly of sorts in a market making in which one market gets a large fraction of the total trading volume. If a financial instrument trades on several locations simultaneously, this "winner takes most" feature results in concentration of trading to the location with the largest number of traders who cannot split their trades.[10]

1.2 Two periods: Observability over time and implications for information transmission.

We have assumed thus far that market makers observe the net order flow only in their own markets and set prices conditional on that information. However, if the private information of informed traders is not publicly revealed for several trading periods, market makers can make more accurate inferences about the innovation in the values of the security by observing past price changes in other markets and inferring trading volumes in those markets.[11]

An interesting economic issue that we examine here is whether competing market makers have incentives to release information publicly about the trading in their markets. We also examine whether regulatory action is likely to be required to achieve information sharing between markets.

We approach this problem by extending our model in the earlier section to allow for more than one round of trading before the private information of the informed trader becomes public knowledge. If informed traders have more than one trading opportunity in which to exploit their private information, then the rapidity with which

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[9] Pagano (1989) also addresses issues relating to concentration of trading on one market when there are fixed costs of entering a particular market. In his model, however, liquidity traders are not allowed to allocate trades across different markets. He examines equilibria when trading is equally costly across markets and when it is not.

[10] In the absence of intertemporal volume effects, such as those discussed in Admati and Pfleiderer (1988), one of the implications of this result would be that when the market with the largest volume is closed for trading, the volume should shift to another market. We believe that in practice, however, the volume and volatility in secondary markets (e.g., the Pacific Exchange) seem to shrink when the primary market (i.e., NYSE/AMEX) is closed (1:00–1:30 P.M. Pacific Time). Similarly, upstairs trading does not seem to increase when the primary market is in the midst of a scheduled closure. This suggests that temporal concentration effects on volume may be significant and may interact with the effects due to concentration across markets. These trading patterns also suggest the existence of an information externality provided by the timely dissemination of price information by the larger markets. We are grateful to Chester Spatt for pointing this out to us.

[11] See also Freedman (1989) and King and Wadhwani (1990).

trading locations share information will materially affect the trading strategies of informed traders.

We stay with the assumption that there is a Bertrand competition *each period* between market makers at each location. This implies that market makers offer zero-profit price schedules *each period*. However, different strategies by market makers, implying different first period λ's, may be consistent with zero profits in each period. The equilibrium will be characterized by market makers in each market choosing strategies that allow them to offer the lowest possible λ's in the *first period*. Any strategy in which a market maker in a given market offers the first period λ that is not the smallest across all possible strategies could not be an equilibrium. This is because another market maker in that market, by choosing an alternative strategy, could attract away all liquidity trading by offering a lower first period λ. A strategy that offers a lower *second period* λ but has a higher *first period* λ would not, in the first period, be attractive to the liquidity traders. Such a strategy could be an equilibrium strategy only under extreme assumptions, such as those that allow market makers to enter into binding multiperiod contracts with liquidity traders. In this model, we do not allow such multiperiod binding contracts.

We show that price competition between market makers at a given location might induce them to transmit trading information to other markets. The intuition is that an informed trader will trade less aggressively in a market in which market makers communicate trading information to other markets, since this adversely affects his expected profits in *all* markets in subsequent periods.

For simplicity, we let the security trade on two markets denoted R and NR. Also, we assume there are two trading periods before the private information possessed by the informed trader becomes public. We assume in this section that there are no large uninformed traders and that no small liquidity traders can move across markets.[12]

We examine the equilibrium under three different scenarios.

Case A. Market makers in neither market release price information about the first period to the other market.

In the first period, market makers in both markets observe the net order flow in their own respective markets and choose the following pricing rules:

$$\Delta P^1_m = \lambda^1_m(x^1_m + u^1_m), \qquad m \in \{R, NR\}.$$

All variables are defined the same way as before: the subscript refers to the market and the superscript refers to the trading period.

[12] We believe that the thrust of our results would not be affected by these simplifications. However, the analysis becomes quite complex and is not undertaken here.

In the second period, the market makers in each of the two markets have information about the past order flow only in their own markets. Thus, their pricing rules are as follows:

$$\Delta P_m^2 = \lambda_m^1(x_m^1 + u_m^1) + \lambda_m^2(x_m^2 + u_m^2), \qquad m \in \{R, NR\}.$$

Case B. Market makers in market R commit to release price information after the first round of trading and market makers in market NR do not.

As in Case A, in the first period, market makers in each market observe the net order flow in their own respective markets and choose the following pricing rules:

$$\Delta P_m^1 = \lambda_m^1(x_m^1 + u_m^1), \qquad m \in \{R, NR\}.$$

In the second period, the market makers in market R have information about the past order flow only in their own markets. Thus, their pricing rule is the following:

$$\Delta P_R^2 = \lambda_R^1(x_R^1 + u_R^1) + \lambda_R^2(x_R^2 + u_R^2).$$

The market makers in the market NR, however, observe ΔP_R^1 and therefore can infer the order flow $(x_R^1 + u_R^1)$. Their pricing rule, in the second period, then, is as follows:

$$\Delta P_{NR}^2 = \lambda_R^{1\prime}(x_R^1 + u_R^1) + \lambda_{NR}^{1\prime}(x_{NR}^1 + u_{NR}^1) + \lambda_{NR}^2(x_{NR}^2 + u_{NR}^2).$$

Case C. Market makers in both the markets commit to release price information after the first round of trading.

As before, in the first period, market makers in each market observe the net order flow in their own respective markets and choose the following pricing rules:

$$\Delta P_m^1 = \lambda_m^1(x_m^1 + u_m^1), \qquad m \in \{R, NR\}.$$

In the second period, market makers in each of the two markets observe the first period prices in the other market and therefore can infer the order flow in the other market. Their pricing rules, in the second period, then, are as follows:

$$\Delta P_m^2 = \lambda_R^{1\prime}(x_R^1 + u_R^1) + \lambda_{NR}^{1\prime}(x_{NR}^1 + u_{NR}^1) + \lambda_m^2(x_m^2 + u_m^2),$$
$$m \in \{R, NR\}.$$

The informed trader in each of the three cases solves the following maximization problem:

$$\max_{\{x_m^t\}_{m=R,NR}^{t=1,2}} E\left[\sum_{t=1,2} \sum_{m=R,NR} x_m^t(v - \Delta P_m^t)\right].$$

The liquidity traders in any given market respond to the *announce-*

ments of λ's by market makers and choose the market maker that offers the lowest λ.

To distinguish the equilibrium values of the λ parameters in the three cases, we will use $\bar{\lambda}$ to denote the equilibrium values in Case A, $\hat{\lambda}$ in Case B, and $\hat{\hat{\lambda}}$ in Case C.

Lemma 2. *If the market makers in market NR do not release price information, then the market makers in market R can offer a lower value of λ in period 1 if they decide to release information than if they do not. Formally,*

$$\bar{\lambda}_R^1 > \hat{\lambda}_R^1.$$

Proof. See the Appendix.

Lemma 3. *If the market makers in market R release price information, then the market makers in market NR can offer a lower value of λ in period 1 if they decide to release information than if they do not. Formally,*

$$\hat{\lambda}_{NR}^1 > \hat{\hat{\lambda}}_{NR}^1.$$

Proof. See the Appendix.

The intuition for these results is straightforward. By committing to release information after the first round of trading, market makers make their own markets less attractive to trade in period 1 for the informed trader since his decision to trade aggressively in the first period adversely affects his expected profits in the second period in *both* markets.

It follows immediately from Lemmas 2 and 3 that neither Case A nor Case B could be an equilibrium. Consider a market maker in Case A. Given that no other market maker commits to release price information, the market maker, by committing to release price information, can offer a first period λ that is between $\hat{\lambda}_R^1$ and $\bar{\lambda}_R^1$. He, therefore, induces all liquidity traders in his market to submit orders to him and thereby makes positive expected profits. A similar argument also rules out Case B as an equilibrium.

We have thus shown the following proposition.

Proposition 4. *If market makers in each market compete to offer the lowest possible λ that is consistent with zero expected profits, then the unique equilibrium is described by Case C in which market makers in both markets commit to release price information after the first round of trading.*

The analysis above indicates that competition for market-making services may enhance information transmission between markets—possibly obviating the need for regulatory action.

1.3 "Cracking down" on insider trading

We now examine whether competing market makers in each market have incentives to *voluntarily* crack down on insider trading or whether regulatory action is likely to be required to achieve that goal. We also examine the claim voiced in the popular press that cracking down on insider trading at one location may drive trading away to other locations.[13]

We approach this issue by adapting the two-period model developed above. The security is taken to trade on two markets denoted D and ND. We assume that arrangements are in place so that trading information at the end of the first round of trading is transmitted both ways between the two markets. As before, we assume that there are only two rounds of trading before the private information possessed by the informed trader becomes public. The informed trader is taken to be an insider (i.e., he gains his private information as a result of insider access).

We wish to analyze the effect of markets putting an insider surveillance system into operation. Certainly if such a system has some, though less than perfect, ability to detect insider trading, it should have the effect of reducing expected insider profits in the market adopting it. We take the expected profits of the informed insider to be reduced by a factor of $(1 - \gamma)$ in the market adopting the system. An interpretation of $(1 - \gamma)$ is that it is the probability with which the trading by the insider is detected to originate from him; the insider's ability to hide the source of the trading will determine the probability with which his trades can be traced. If it is detected that the insider was responsible for certain trades, he is compelled to pay fines equal to the profits made. It is clear that even if the fines were a multiple of the profits made, the expected profits of the insider could be expressed in a similar linear fashion.[14] The expected profits in the other market are not directly affected as a result of the monitoring system; to the extent that the adoption of the system affects trading strategies, the expected profits of the insider in each market will, naturally, be affected. Under these assumptions, the informed insider will solve the following maximization problem:

[13] See references cited in note 4.

[14] There are other ways of formalizing the penalties suffered by the insider. One alternative is to prevent the insider from trading once he/she is caught: this approach does not seem realistic since the time period spanned by the two periods in our model is of the order of the time insider's information remains private, which is generally much shorter than the average time required to detect an instance of insider trading and to pursue conviction.

$$\max_{\{x_m^t\}_{m=D,ND}^{t=1,2}} E\left[\sum_{t=1}^{2} \gamma_D x_D^t(v - \Delta P_D^t) + \sum_{t=1}^{2} \gamma_{ND} x_{ND}^t(v - \Delta P_{ND}^t)\right].$$

As in the earlier section, we examine the equilibrium under three different scenarios.

Case A. Market makers in neither market institute the insider surveillance system. In this case,

$$\gamma_D = \gamma_{ND} = 1.$$

Case B. Market makers in market *D* institute the insider surveillance system but market makers in market *ND* do not. In this case,

$$0 < \gamma_D = \gamma < 1, \qquad \gamma_{ND} = 1.$$

Case C. Market makers in both markets institute the insider surveillance system. In this case,

$$0 < \gamma_D = \gamma_{ND} = \gamma < 1.$$

To distinguish the equilibrium values of the λ parameters in the three cases, we will use $\bar{\lambda}$ to denote the equilibrium values in Case A, $\hat{\lambda}$ in Case B, and $\hat{\hat{\lambda}}$ in Case C.

Lemma 4. *If the market makers in market ND do not institute the insider surveillance system, then the market makers in market D can offer a lower value of λ in period 1 if they decide to institute the insider surveillance system than if they do not. Formally,*

$$\bar{\lambda}_D^1 > \hat{\lambda}_D^1.$$

Proof. See the Appendix.

Lemma 5. *If the market makers in market D institute the insider surveillance system, then the market makers in market ND can offer a lower value of λ in period 1 if they decide to institute the insider surveillance system than if they do not. Formally,*

$$\hat{\lambda}_{ND}^1 > \hat{\hat{\lambda}}_{ND}^1.$$

Proof. See the Appendix.

The intuition for these results is straightforward. By instituting the insider surveillance system, market makers make their own markets less attractive to trade in period 1 for the informed trader since his decision to trade aggressively in the first period adversely affects his expected profits in the second period in *both* markets.

We have thus shown the following proposition.

Proposition 5. *If the market makers compete to offer the lowest possible λ that is consistent with zero-expected profits, then the unique equilibrium is described by Case C in which both markets choose to institute the insider surveillance system.*

Since market makers have incentives to institute insider surveillance systems *voluntarily,* we conclude that regulatory action may not be required to achieve that goal.[15] Competition for market-making services would induce market makers to run "clean markets." As a result of this desire to project a clean image, market makers may even choose to cooperate with regulatory agencies such as the SEC.

Lemma 6. *If market makers in market D institute the insider surveillance system and market makers in market ND do not, then market makers in market D will be able to offer lower value first period λ's than those offered by market makers in market ND if the variance of the liquidity trading was identical across both markets and both time periods. Formally,*

$$(\sigma_m^t)^2 = \sigma_0^2, \qquad m \in \{D, ND\},\ t \in \{1, 2\}$$
$$\Rightarrow \hat{\lambda}_D^1 < \hat{\lambda}_{ND}^1, \qquad \hat{\lambda}_D^2 = \hat{\lambda}_{ND}^2.$$

Proof. See the Appendix.

We observe that the adverse selection costs for liquidity traders are lower in the market where market makers actively attempt to counter insider trading. The intuition for the result is as follows. Trading in the first period in either market affects the total expected profits in the second period, since trading by the insider partially reveals his private information to both markets. If the expected profits in the first market are reduced on account of monitoring, the insider will trade less aggressively in the first period in the first market compared to the other market. The second period, however, is the last period of trading before the information becomes public. Hence, given that market makers in both markets have common information before receiving the second period order flow, the insider's trading strategies in the second period will be the same in the two markets.[16]

[15] See references cited in note 3.

[16] Throughout this analysis, we assumed that there was a single informed insider. Our main results, however, are not affected by this simplifying assumption. When there are more than one informed insider, then—as is pointed out in Admati and Pfleiderer (1988)—they compete with each other, which helps the liquidity traders. Assume a single informed insider is analytically equivalent—for our results—to an exogenously fixed number of informed insiders. It is true that an *exogenous* reduction in the number of informed insiders would result in larger *overall* losses by the liquidity traders to the informed insiders, but our result that the market in which market makers crack down on the insiders will have lower first period λ's would still be true. If we were to make the number of informed insiders endogenous and the fines collected from the detected insiders were redistributed to the market makers, then even the *overall* losses by the liquidity traders to the informed

We assumed in Lemma 6 that the size of liquidity trading was exogenous and identical across both markets. If some of the small liquidity traders have discretion to choose the market in which adverse selection costs are lower, then the result in this Lemma is further reinforced since all these traders will concentrate on market D (from Proposition 3), reducing the first period λ's even further. This, in turn, would attract more trading by the large liquidity trader as well as by the insider. This is contrary to the speculation in the popular press that cracking down on insider trading at one location may drive trading away to other locations.

2. Conclusion

We have investigated some of the issues pertaining to the trading of a security at multiple locations simultaneously. Information lags between different trading centers produce short-term disparities in the prices at which the security trades at different locations at any given time. While the formal analysis is done assuming that an identical security trades at multiple locations, the ideas developed in the article should apply more broadly to nonidentical securities that are affected by common factors such as stocks and their derivative securities, forward and future contracts.

We show that competition for market-making services induces market makers to take actions, such as making the price information public and cracking down on insider trading, that deter informed trading. This indicates that even in the absence of government regulation, competitive economic forces alone might facilitate both the transmission of information between market locations and deterrence of insider trading. These insights may provide a starting point in understanding the design of security exchanges.

One of the limitations of the approach in this article is that we have not explicitly modeled competition between exchanges. A fruitful avenue of research may be to view exchanges as being strategic players attempting to maximize some objective. For example, if traders pay transaction fees to the exchange, then it may be reasonable to view the exchange as attempting to maximize the revenue it earns over time as a function of trading volume. Competition among exchanges, then, would get reflected in the profits of the exchange and in the pricing of securities.

would be smaller, even though there would be an endogenous reduction in the number of informed insiders in equilibrium. Further, following Fishman and Hagerty (1990), if we were to make a distinction between informed insiders and informed noninsiders, then a system that discourages entry by informed insiders may encourage entry by informed noninsiders; and since all informed traders compete, the liquidity traders are made better off even further.

If the exchanges are viewed as being strategic players, the assumption of perfect price competition each period may not be appropriate. By releasing price information an exchange can offer a lower cost of trading in the earlier trading rounds. This is the reason emphasized in this article for competitive market makers to disclose current price information. However, price disclosure has another important effect that is not emphasized in this article—namely, that the market attracting the bulk of the order flow seems to subsidize other markets by timely dissemination of price information.[17] If an exchange is acting strategically, it may be able to offer a lower cost of trading in later periods by withholding price information in earlier trading rounds. The exchange, therefore, faces an intertemporal trade-off in deciding whether or not to release price information. The price disclosure issue, then, becomes more complex than in the scenario modeled in this article.

We leave these issues for future research.

Appendix

Proof of Lemma 1

Substituting for λ_i from (4) and for k_i from (3) into (5), and simplifying we get

$$c_i = \frac{\Sigma_{i=1}^N c_i}{\left[(\Sigma_{i=1}^N c_i)^2 - \sigma_d^2/\sigma_v^2\right]^{1/2}} \frac{\sigma_i}{\sigma_v}. \tag{A1}$$

Summing the above over i and simplifying, we get

$$\left(\sum_{i=1}^N c_i\right)^2 = \frac{\sigma_d^2 + (\Sigma_{i=1}^N \sigma_i)^2}{\sigma_v^2} = \frac{\sigma^2}{\sigma_v^2}. \tag{A2}$$

Substituting back in (A1) we obtain the desired expression for c_i. Then, expressions for λ_i and k_i are obtained by direct substitutions into (4) and (3). Since all parameters λ_i, c_i, and k_i are positive, the second-order conditions are also satisfied. ■

Proof of Proposition 1

Substituting $\lambda_i = 1/(2c_i)$ into (5), we get

$$c_i^2\sigma_v^2 + \sigma_i^2 + k_i^2\sigma_d^2 = 2c_i^2\sigma_v^2, \qquad \forall i, \tag{A3}$$

$\rho(x_i^* + u_i + k_i d,\ x_j^* + u_j + k_j d)$

[17] For example, the price-setting procedures used by the Pacific Stock Exchange for NYSE stocks and the pricing by competing dealers rely heavily upon the NYSE pricing. At times when this externality is not present, because there is no trading on the NYSE, trading in subsidiary markets seems to be reduced (see note 10).

$$= \frac{\mathrm{Cov}\left[c_i v + u_i + k_i d,\ c_j v + u_j + k_j d\right]}{\left[\mathrm{Var}(c_i v + u_i + k_i d)\right]^{1/2}\left[\mathrm{Var}(c_j v + u_j + k_j d)\right]^{1/2}}$$

$$= \frac{c_i c_j \sigma_v^2 + k_i k_j \sigma_d^2}{(c_i^2\sigma_v^2 + \sigma_i^2 + k_i^2\sigma_d^2)^{1/2}(c_j^2\sigma_v^2 + \sigma_j^2 + k_j^2\sigma_d^2)^{1/2}} \qquad \text{[from (A3)]}$$

$$= \frac{1}{2}\left[1 + \frac{\sigma_d^2}{\sigma^2}\right] \qquad \text{[from (3), (A2)].} \qquad \text{(A4)}$$

■

Proof of Proposition 2

Substituting from (6)–(9) in (1), we get

$$\Delta P_i = \frac{1}{2}v + \frac{1}{2}\sigma_v\left[\frac{1}{\sigma}d + \frac{1}{\sigma}\frac{(\Sigma_{i=1}^N \sigma_i)}{\sigma_i}u_i\right], \qquad \forall i,$$

$$\mathrm{Var}[\Delta P_i] = \frac{1}{4}\sigma_v^2 + \frac{1}{4}\sigma_v^2\left[\frac{1}{\sigma^2}\sigma_d^2 + \frac{1}{\sigma^2}\frac{(\Sigma_{i=1}^N \sigma_i)^2}{\sigma_i^2}\sigma_i^2\right] = \frac{1}{2}\sigma_v^2. \qquad \text{(A5)}$$

From (A4) and (A5), we get

$$\mathrm{Cov}[\Delta P_i, \Delta P_j] = \tfrac{1}{4}\sigma_v^2(1 + \sigma_d^2/\sigma^2), \qquad \forall i,j,\ i \neq j. \qquad \text{(A6)}$$

From (A3), we have

$$\sigma_j^2 = c_j^2\sigma_v^2 - k_j^2\sigma_d^2 = c_j^2\sigma_v^2\left[1 - \sigma_d^2/\sigma^2\right] \qquad \text{[from (3), (A2)].} \qquad \text{(A7)}$$

Because of normality, we have the following multiple regression:

$$v = \sum_{i=1}^{N} \lambda_i'(c_i v + u_i + k_i d) + \epsilon. \qquad \text{(A8)}$$

Multiplying by $(c_i v + u_i + k_i d)$, taking expectations, and manipulating, we get

$$c_j\sigma_v^2 = c_j\sigma_v^2\sum_{i=1}^{N}\lambda_i' c_i + \lambda_j' c_j^2\sigma_v^2\left[1 - \frac{\sigma_d^2}{\sigma^2}\right] + c_j\sigma_v^2\frac{\sigma_d^2}{\sigma^2}\sum_{i=1}^{N}\lambda_i' c_i$$

[from (3), (A7), and (A2)].

Dividing both sides by $c_j\sigma_v^2$ and summing over all j and solving, we get

$$\sum_{i=1}^{N}\lambda_i' c_i = \frac{N}{(N+1) + (\sigma_d^2/\sigma^2)(N-1)}$$

$$\Rightarrow \frac{\lambda_i'}{\lambda_i} = \frac{2}{(N+1) + (\sigma_d^2/\sigma^2)(N-1)} \equiv \phi, \qquad \forall i; \qquad \text{(A9)}$$

$$\Delta P = E\left[v \mid \{x_i^* + u_i + k_i d\}_{i=1}^{N}\right] = \sum_{i=1}^{N} \lambda_i'(x_i^* + u_i + k_i d)$$

$$= \phi \sum_{i=1}^{N} \Delta P_i \qquad \text{[from (A8) and (A9)]};$$

$\operatorname{Var}\left[\tilde{v} \mid \{P_i\}_{i=1}^{N}\right] = \operatorname{Var}\left[\tilde{v} \mid \{x_i^* + u_i + k_i d\}_{i=1}^{N}\right] = \operatorname{Var}\left[\Delta P\right].$

Therefore,

$$\psi \equiv \frac{\operatorname{Var}\left[\Delta P\right]}{\sigma_v^2}$$

$$= \frac{\phi^2\left[N \operatorname{Var}\left[\Delta P_i\right] + N(N-1)\operatorname{Cov}\left[\Delta P_i, \Delta P_j\right]\right]}{\sigma_v^2}$$

$$= \frac{N}{(N+1) + (\sigma_d^2/\sigma^2)(N-1)} \qquad \text{[from (14) and (15)]}.$$

Proofs of Lemmas 2 and 3
Let $c_m^1 \equiv x_m^{1*}/v$, for $m \in \{R, NR\}$.

Case A. Here the markets are informationally separate. For market R (and similarly for NR), we first solve the informed trader's maximization problem in the second period. Substituting the first-order condition so obtained into the informed trader's first period maximization problem, we get the following first-order condition:

$$\left[1 - \frac{\lambda_R^1}{2\lambda_R^2}\right] - c_R^1\left[2\lambda_R^1 - \frac{(\lambda_R^1)^2}{2\lambda_R^2}\right] = 0. \qquad \text{(A10)}$$

Imposing the conditions that the market makers set prices equal to expectations of v conditional on the price and order flow information available to them, we get the following restrictions:

$$\lambda_R^1\left[(c_R^1)^2\sigma_v^2 + (\sigma_R^1)^2\right] = c_R^1\sigma_v^2, \qquad \text{(A11)}$$

$$4(\lambda_R^2)^2(\sigma_R^2)^2 = (1 - \lambda_R^1 c_R^1)\sigma_v^2. \qquad \text{(A12)}$$

Let $\bar{c}_R^1$ and $\bar{\lambda}_R^1$ denote the equilibrium values that satisfy Equations (A10)–(A12) and the second-order conditions. The second-order conditions rule out unbounded destabilizing strategies by traders, as in Kyle (1985).

Case B. In this case, the market makers in market R are committed to releasing information at the end of the first period of trading, while the market makers in NR are not. The informed trader's information set in the second period contains all the prices (and therefore the order flows) from the first period. As in Case A, backward optimization of the informed trader's maximization problem yields the following first-order conditions:

$$\left[1 - \frac{\lambda_R^1}{2\lambda_R^2} - \frac{\lambda_R^{1'}}{2\lambda_{NR}^2}\right] - c_R^1\left[2\lambda_R^1 - \frac{(\lambda_R^1)^2}{2\lambda_R^2} - \frac{(\lambda_R^{1'})^2}{2\lambda_{NR}^2}\right]$$

$$+ c_{NR}^1\frac{\lambda_R^{1'}\lambda_{NR}^{1'}}{2\lambda_{NR}^2} = 0, \tag{A13}$$

$$\left[1 - \frac{\lambda_{NR}^{1'}}{2\lambda_{NR}^2}\right] - c_{NR}^1\left[2\lambda_{NR}^1 - \frac{(\lambda_{NR}^{1'})^2}{2\lambda_{NR}^2}\right] + c_R^1\frac{\lambda_R^{1'}\lambda_{NR}^{1'}}{2\lambda_{NR}^2} = 0. \tag{A14}$$

Imposing the price-setting conditions by market makers we get the following:

$$\lambda_R^1\left[(c_R^1)^2\sigma_v^2 + (\sigma_R^1)^2\right] = c_R^1\sigma_v^2, \tag{A15}$$

$$\lambda_{NR}^1\left[(c_{NR}^1)^2\sigma_v^2 + (\sigma_{NR}^1)^2\right] = c_{NR}^1\sigma_v^2, \tag{A16}$$

$$\lambda_R^{1'}\left[(c_R^1)^2\sigma_v^2 + (\sigma_R^1)^2\right] + \lambda_{NR}^{1'}c_R^1c_{NR}^1\sigma_v^2 = c_R^1\sigma_v^2, \tag{A17}$$

$$\lambda_R^{1'}c_R^1c_{NR}^1\sigma_v^2 + \lambda_{NR}^{1'}\left[(c_{NR}^1)^2\sigma_v^2 + (\sigma_{NR}^1)^2\right] = c_{NR}^1\sigma_v^2, \tag{A18}$$

$$4(\lambda_R^2)^2(\sigma_R^2)^2 = (1 - \lambda_R^1c_R^1)\sigma_v^2, \tag{A19}$$

$$4(\lambda_{NR}^2)^2(\sigma_{NR}^2)^2 = (1 - \lambda_R^{1'}c_R^1 - \lambda_{NR}^{1'}c_{NR}^1)\sigma_v^2. \tag{A20}$$

We denote the equilibrium values that satisfy Equations (A13)–(A20) and the second-order conditions by $\hat{c}_R^1$, $\hat{\lambda}_R^1$ and $\hat{c}_{NR}^1$, $\hat{\lambda}_{NR}^1$.

Case C. The first-order conditions for the maximization problem are

$$\left[1 - \frac{\lambda_R^{1'}}{2\lambda_R^2} - \frac{\lambda_R^{1'}}{2\lambda_{NR}^2}\right] - c_R^1\left[2\lambda_R^1 - \frac{(\lambda_R^{1'})^2}{2\lambda_R^2} - \frac{(\lambda_R^{1'})^2}{2\lambda_{NR}^2}\right]$$

$$+ c_{NR}^1\left[\frac{\lambda_R^{1'}\lambda_{NR}^{1'}}{2\lambda_R^2} + \frac{\lambda_R^{1'}\lambda_{NR}^{1'}}{2\lambda_{NR}^2}\right] = 0, \tag{A21}$$

$$\left[1 - \frac{\lambda_{NR}^{1'}}{2\lambda_{NR}^2} - \frac{\lambda_{NR}^{1'}}{2\lambda_R^2}\right] - c_{NR}^1\left[2\lambda_{NR}^1 - \frac{(\lambda_{NR}^{1'})^2}{2\lambda_{NR}^2} - \frac{(\lambda_{NR}^{1'})^2}{2\lambda_R^2}\right]$$

$$+ c_R^1\left[\frac{\lambda_R^{1'}\lambda_{NR}^{1'}}{2\lambda_{NR}^2} + \frac{\lambda_R^{1'}\lambda_{NR}^{1'}}{2\lambda_R^2}\right] = 0. \tag{A22}$$

Market makers' price-setting restrictions are identical to those in (A15)–(A20) except for (A19), which is replaced by the following:

$$4(\lambda_R^2)^2(\sigma_R^2)^2 = (1 - \lambda_R^{1\prime} ct_R^1 - \lambda_{NR}^{1\prime} c_{NR}^1)\sigma_v^2. \tag{A23}$$

We denote the equilibrium values that satisfy these equations and the second-order conditions by $\hat{c}_R^1$, $\hat{\lambda}_R^1$ and $\hat{\hat{c}}_{NR}^1$, $\hat{\hat{\lambda}}_{NR}^1$.

Proof of Lemma 2. First observe that

$$-\frac{\lambda_R^{1\prime}}{2\lambda_{NR}^2} - c_R^1\left[-\frac{(\lambda_R^{1\prime})^2}{2\lambda_{NR}^2}\right] + c_{NR}^1\frac{\lambda_R^{1\prime}\lambda_{NR}^{1\prime}}{2\lambda_{NR}^2} < 0,$$

since from (A20) $(1 - c_R^1\lambda_R^{1\prime} - c_{NR}^1\lambda_{NR}^{1\prime}) > 0$. Substituting in (A13) we get

$$\left[1 - \frac{\lambda_R^1}{2\lambda_R^2}\right] - c_R^1\left[2\lambda_R^1 - \frac{(\lambda_R^1)^2}{2\lambda_R^2}\right]$$
$$= \left[1 - \frac{\lambda_R^1}{2\lambda_R^2}\right]\left[1 - c_R^1\lambda_R^1\right] - c_R^1\lambda_R^1 > 0. \tag{A24}$$

$\hat{\lambda}_R^1$ satisfies (A24), (A11), and (A12). Observe that the set of equations determining $\bar{\lambda}_R^1$ and $\hat{\lambda}_R^1$ are identical except that the expression in (A23) is greater that zero. We know that c_R^1 is increasing in λ_R^1 [from Equation (A15)], while λ_R^2 is decreasing in c_R^1 [from Equation (A19)]. The expression in (A24) is decreasing in c_R^1. Hence, for the inequality (A24) to be satisfied, it must be the case that $\hat{c}_R^1 < \bar{c}_R^1$. It follows that $\hat{\lambda}_R^1 < \bar{\lambda}_R^1$. ■

For the proofs of Lemmas 3–5, it is convenient to define two functions:

$$g(c_R^1, c_{NR}^1) \equiv \left[1 - \frac{\lambda_R^{1\prime}}{2\lambda_R^2} - \frac{\lambda_R^{1\prime}}{2\lambda_{NR}^2}\right] - c_R^1\left[2\lambda_R^1 - \frac{(\lambda_R^{1\prime})^2}{2\lambda_R^2} - \frac{(\lambda_R^{1\prime})^2}{2\lambda_{NR}^2}\right]$$
$$+ c_{NR}^1\left[\frac{\lambda_R^{1\prime}\lambda_{NR}^{1\prime}}{2\lambda_R^2} + \frac{\lambda_R^{1\prime}\lambda_{NR}^{1\prime}}{2\lambda_{NR}^2}\right],$$

$$f(c_R^1, c_{NR}^1) \equiv \left[1 - \frac{\lambda_{NR}^{1\prime}}{2\lambda_{NR}^2} - \frac{\lambda_{NR}^{1\prime}}{2\lambda_R^2}\right] - c_{NR}^1\left[2\lambda_{NR}^1 - \frac{(\lambda_{NR}^{1\prime})^2}{2\lambda_{NR}^2} - \frac{(\lambda_{NR}^{1\prime})^2}{2\lambda_R^2}\right]$$
$$+ c_R^1\left[\frac{\lambda_R^{1\prime}\lambda_{NR}^{1\prime}}{2\lambda_{NR}^2} + \frac{\lambda_R^{1\prime}\lambda_{NR}^{1\prime}}{2\lambda_R^2}\right].$$

We substitute the restrictions for Case C, (A15)–(A18), (A20), and (A23), in the $g(\cdot)$ and $f(\cdot)$ functions so as to be able to express these functions solely in terms of c_R^1, c_{NR}^1 and λ_R^1, λ_{NR}^1. Since λ_R^1 is determined only by c_R^1 [by Equation (A15)] and λ_{NR}^1 only by c_{NR}^1, we essentially have two functions in two unknowns.

On substitution of the constraints, and rearranging, we can express the functions as

$$g(\cdot) = 1 - 2c_R^1\lambda_R^1 - (\lambda_R^1)(1 - c_R^1\lambda_R^1)^{1/2} \times \left[\frac{1 - \lambda_{NR}^1 c_{NR}^1}{1 - \lambda_{NR}^1 c_{NR}^1 \lambda_R^1 c_R^1}\right]^{3/2} k,$$

where $k = (\sigma_R^2 + \sigma_{NR}^2)/\sigma_v$. Similarly, we can express

$$f(\cdot) = 1 - 2c_{NR}^1\lambda_{NR}^1 - (\lambda_{NR}^1)(1 - c_{NR}^1\lambda_{NR}^1)^{1/2} \times \left[\frac{1 - \lambda_R^1 c_R^1}{1 - \lambda_{NR}^1 c_{NR}^1 \lambda_R^1 c_R^1}\right]^{3/2} k.$$

$g(\cdot)$ is increasing in c_{NR}^1 and decreasing in c_R^1, and $f(\cdot)$ is increasing in c_R^1 and is decreasing in c_{NR}^1.

Consider the curves $g(\cdot) = 0$ and $f(\cdot) = 0$ defined in the (c_R^1, c_{NR}^1) plane. Both these curves are monotonically increasing. The intersection of these two curves gives the equilibrium values of c_R^1, c_{NR}^1 in Case C.

We also define a third function:

$$h(c_R^1, c_{NR}^1) \equiv \frac{(1 - 2\lambda_R^1 c_R^1)}{\lambda_R^1(1 - c_R^1\lambda_R^1)^{1/2}\left(\frac{1 - \lambda_{NR}^1 c_{NR}^1}{1 - \lambda_{NR}^1 c_{NR}^1 \lambda_R^1 c_R^1}\right)^{3/2}} \times \frac{(1 - 2\lambda_{NR}^1 c_{NR}^1)}{\lambda_{NR}^1(1 - c_{NR}^1\lambda_{NR}^1)^{1/2}\left(\frac{1 - \lambda_R^1 c_R^1}{1 - \lambda_{NR}^1 c_{NR}^1 \lambda_R^1 c_R^1}\right)^{3/2}}.$$

The function $h(\cdot)$ is decreasing both in c_R^1 and in c_{NR}^1. In the (c_R^1, c_{NR}^1) plane, the equation $h(\cdot) = k^2$ defines a downward sloping curve. In the equilibrium for Case C, we have $g(\hat{\hat{c}}_R^1, \hat{\hat{c}}_{NR}^1) = 0$, $f(\hat{\hat{c}}_R^1, \hat{\hat{c}}_{NR}^1) = 0$, and $h(\hat{\hat{c}}_R^1, \hat{\hat{c}}_{NR}^1) = k^2$.

Proof outline for Lemma 3. We can show that the equilibrium in Case B is such that $f(\hat{c}_R^1, \hat{c}_{NR}^1) < 0$ and $h(\hat{c}_R^1, \hat{c}_{NR}^1) < k^2$.

In the (c_R^1, c_{NR}^1) plane, $f(\cdot) < 0$ describes a region to the left of the curve defined by $f(\cdot) = 0$. Similarly, $h(\cdot) < k^2$ describes a region above the curve defined by $h(\cdot) = k^2$. Since $f(\cdot)$ is upward sloping and $h(\cdot)$ is downward sloping, the intersection of the curves that defines the equilibrium in Case B will have $\hat{c}_{NR}^1 > \hat{\hat{c}}_{NR}^1$, which is equivalent to showing that $\hat{\lambda}_{NR}^1 > \hat{\hat{\lambda}}_{NR}^1$. ■

Proof outlines for Lemmas 4 and 5
Since the maximand for the maximization problem in Case C is simply γ multiplied by the maximand for the maximization problem in Case A, the equilibrium values of λ parameters and c parameters in Cases A and C will be identical.

Cases A and C. The first-order conditions from the informed trader's maximization problem as well as the market makers' price-setting restrictions are identical to those in Case C in Section 1.2, with c_D^1 replacing c_R^1 and c_{ND}^1 replacing c_{NR}^1. We denote the equilibrium values in these cases by $\hat{c}_D^1$, $\hat{\lambda}_D^1$ and $\hat{c}_{ND}^1$, $\hat{\lambda}_{ND}^1$.

Case B. The only change from Cases A and C above is in the first-order conditions, which are given by

$$\left[\gamma - \frac{\gamma\lambda_D^{1'}}{2\lambda_D^2} - \frac{\lambda_D^{1'}}{2\lambda_{ND}^2}\right] - c_D^1\left[2\gamma\lambda_D^1 - \frac{\gamma(\lambda_D^{1'})^2}{2\lambda_D^2} - \frac{(\lambda_D^{1'})^2}{2\lambda_{ND}^2}\right] + c_{ND}^1\left[\frac{\gamma\lambda_D^{1'}\lambda_{ND}^{1'}}{2\lambda_D^2} + \frac{\lambda_D^{1'}\lambda_{ND}^{1'}}{2\lambda_{ND}^2}\right] = 0, \tag{A25}$$

$$\left[1 - \frac{\lambda_{ND}^{1'}}{2\lambda_{ND}^2} - \frac{\gamma\lambda_{ND}^{1'}}{2\lambda_D^2}\right] - c_{ND}^1\left[2\gamma\lambda_{ND}^1 - \frac{(\lambda_{ND}^{1'})^2}{2\lambda_{ND}^2} - \frac{\gamma(\lambda_{ND}^{1'})^2}{2\lambda_D^2}\right] + c_D^1\left[\frac{\lambda_D^{1'}\lambda_{ND}^{1'}}{2\lambda_{ND}^2} + \frac{\gamma\lambda_D^{1'}\lambda_{ND}^{1'}}{2\lambda_D^2}\right] = 0. \tag{A26}$$

Market makers' price-setting restrictions are identical to those in Cases A and C. We denote the equilibrium values in this case by $\hat{c}_D^1$, $\hat{\lambda}_D^1$ and $\hat{c}_{ND}^1$, $\hat{\lambda}_{ND}^1$.

Proof outline for Lemma 4. Here we use the same functions $f(\cdot)$, $g(\cdot)$, and $h(\cdot)$ as before except that c_D^1 replaces c_R^1 and c_{ND}^1 replaces c_{NR}^1. The equilibrium in Case A is such that $g(\bar{c}_D^1, \bar{c}_{ND}^1) = 0$, $h(\bar{c}_D^1, \bar{c}_{ND}^1) = k^2$. We can show that the equilibrium in Case B is such that $g(\hat{c}_D^1, \hat{c}_{ND}^1) > 0$ and $h(\hat{c}_D^1, \hat{c}_{ND}^1) > 0$. In the (c_D^1, c_{ND}^1) plane, $h(\cdot) > k^2$ defines a region that is below that defined by the curve $h(\cdot) = k^2$. $g(\cdot) > 0$ defines a region that is to the left of the curve defined by $g(\cdot) = 0$. Since $h(\cdot)$ is downward sloping and $g(\cdot)$ is upward sloping, the intersection of the curves that defines the equilibrium in Case B will have $\hat{c}_D^1 < \bar{c}_D^1$, which is equivalent to showing that $\hat{\lambda}_D^1 < \bar{\lambda}_D^1$. ■

Proof outline for Lemma 5. The first step is to show that at the point of intersection, the curves defined by the functions $g(\cdot) = 0$ and $f(\cdot) = 0$ are such that the slope of curve defined by $g(\cdot) = 0$ is greater than the slope of the curve defined by $f(\cdot) = 0$ in the (c_D^1, c_{ND}^1) plane. Recall that the curves defined by $g(\cdot)$, $f(\cdot)$ are upward sloping. Now for a fixed value of c_{ND}^1 that is below the intersection point of the curves defined by these functions, the corresponding value of c_D^1 must be less for the curve defined by $g(\cdot)$ than for the curve defined by $f(\cdot)$. Similarly, for a fixed value of c_{ND}^1 that is above the intersection point of the curves defined by these functions, the corresponding value of c_D^1 must be greater for the curve defined by $g(\cdot)$ than for the curve defined by $f(\cdot)$. To determine the effect of γ decreasing below 1 on the equilibrium value of c_{ND}^1, we evaluate the effect on the value of c_D^1 for the two curves defined by the functions $f(\cdot)$ and $g(\cdot)$, keeping the value of c_{ND}^1 fixed at the equilibrium value for $\gamma = 1$. It is shown that as γ decreases below 1, the change in c_D^1 is such as to imply that the equilibrium value of c_{ND}^1, and, therefore, the corresponding value of λ_{ND}^1 rises. ■

Proof of Lemma 6

Define new λ^* variables corresponding to each λ variable as $\lambda^* \equiv \hat{\lambda}\sigma_0/\sigma_v$ and similarly define $c^* \equiv \hat{c}\sigma_v/\sigma_0$ and substitute in the restrictions for Cases A and C of Section 2.3. Substituting and rearranging, we get

$$2\left[\frac{(c_{ND}^{1*})^2}{1 + (c_{ND}^{1*})^2} - \frac{(c_D^{1*})^2}{1 + (c_D^{1*})^2}\right] + \frac{(\gamma + 1)}{\left[1 + (c_D^{1*})^2 + (c_{ND}^{1*})^2\right]^{1/2}}\left[c_{ND}^{1*} - c_D^{1*}\right]$$
$$= (1 - \gamma)\left[1 - \frac{2(c_D^{1*})^2}{1 + (c_D^{1*})^2}\right].$$

The right-hand side of the above equation is positive. If $c_D^{1*} \geq c_{ND}^{1*}$, the left-hand side would not be positive, which is not possible. Therefore, it follows that $c_D^{1*} < c_{ND}^{1*}$. Since λ_D^{1*} is increasing in c_D^{1*} and λ_{ND}^{1*} is increasing in c_{ND}^{1*}, we get the result that $\lambda_D^{1*} < \lambda_{ND}^{1*}$, which implies that $\hat{\lambda}_D^1 < \hat{\lambda}_{ND}^1$. ■

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