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Econ 201B - Zame

GAME THEORY - HW #3

1. (a) The seller types are $T_S = \{g, b\}$. There are no buyer types. The strategies for the buyer are $\{AA, AR, RA, RR\}$, where AR means accept high, reject low, etc. For the seller, the strategies are $\{HH, HL, LH, LL\}$, where HL means high price for good car, low for bad, etc.

   (b) We compute the payoff matric for the pure strategies:

	AA	AR	RA	RR
HH	**4.5,3**	**4.5,3**	0,0	0,0
HL	3.5,4	3,3	0.5,1	0,0
LH	1.5,6	1.5,0	0,6	0,0
LL	0.5,7	0.5,0	**0.5,7**	0,0

   The bold numbers represent weakly dominant strategies. So the pure NE strategies are:

   i. (HH,AA)
   ii. (HH,AR)
   iii. (LL,RA)

   Since the first two strategies yield the same utility, we also have mixed strategy NE by combining them. In behavioral form, the strategies are

   i. $\sigma_S(H|g) = \sigma_S(H|b) = 1;$
   $\sigma_B(A|H) = 1,$
   $\sigma_B(A|L) = \alpha \quad \text{for } \alpha \in [0, 1]$

   ii. $\sigma_S(H|g) = \sigma_S(H|b) = 0;$
   $\sigma_B(A|H) = 0,$
   $\sigma_B(A|L) = 0$

2. The above analysis holds for any fraction of good cars, and in each NE the car is always sold.

3. (a) We have both ignorant and knowledgable buyers. Additionally, the knowledgable buyer gets a signal about the quality of the car. We therefore have three types, denoted $T_B = \{i, kg, kb\}$. The strategies and seller types are left unchanged.

(b) Knowledgable buyers cannot increase utility by deviating from the strategies in Problem 1. Thus, the NE we found above are still NE.

Note, however, that knowledgable buyers who observe a bad car are indifferent between the strategies AA, AR, and RA. If they choose RA, then sellers gain by deviating from HH to HL. In this problem, then gain (.5) from deviating with the knowledgable buyers playing RA is not sufficient to offset the cost of deviating against the ignorant buyers playing AA (-1). If the percentage of knowledgable buyers were large enough, however, an additional NE would arise.

(c) No. The equilibria are unchanged, so the outcome are also unchanged.

4. (a) In a symmetric equilibrium, each resident has the same probability q of reporting the crime. In equilibrium, q is chosen so as to make each resident indifferent between reporting and not.

$$u(R) = 1 - \varepsilon$$
$$u(N) = 1 - (1-q)^{N-1}$$

We set $u(R) = u(N)$ and solve for q to obtain

$$q = 1 - \varepsilon^{\frac{1}{N-1}}$$

(b) $P(R) = 1 - (1-q)^N = 1 - \varepsilon^{\frac{N}{N-1}}$

(c) Taking limits in (a) and (b), we find $q \to 0$ and $P(R) \to 1 - \varepsilon$. That is, the chance of any individual reporting falls to zero, but the chance of someone reporting converges.

5. (a) The probability of another resident reporting now changes from q to pq, so

$$u(N) = 1 - (1-pq)^{\frac{1}{N-1}}$$

Again setting $u(R) = r(N)$, we get

$$q = \frac{1}{p}\left[1 - \varepsilon^{\frac{1}{N-1}}\right]$$

(b) Each resident now has a pq chance of reporting, so the cumulative probability is as in the previous problem,

$$P(R) = 1 - (1 - pq)^N = 1 - \varepsilon^{\frac{N}{N-1}}$$

(c) As $p \to 1$, our expressions for q and $P(R)$ approach their values from Problem 4. As $p \to 0$, $q \to 1$ so as long as anyone sees the vandalism, they are certain to report it.

(d) Taking limits with p fixed, we obtain $q \to 0$ and $P(R) \to 1 - \varepsilon$.

(e) The limit as $p \to 0$ and $N \to \infty$ is not well-defined. The order in which we take the limits changes the result. If we take the p-limit first, we get guaranteed reporting as in (c), but if we take the N-limit first, we get the result from (d).

6. (a) Each player has the same actions and types:

$$A_i = \{B, P\}$$
$$T_i = \{\text{AA,AJ,JJ}\}$$

We may specify the behavioral strategies by a triple (p_1, p_2, p_3) indicating the probability of betting with AA, AJ, JJ respectively. Formally, the strategies are all the mappings σ_i from T_i onto $[0, 1]$.

(b) We can find the (unique) NE strategy by considering the best responses to a generic strategy (p_1, p_2, p_3). When a player holds AA, betting stongly dominates passing, regardless of the opponent's strategy. Thus, any NE strategy must have $p_1 = 1$.

Suppose then, that our opponent plays $(1, p_2, p_3)$. Then if we hold AJ, then we can calculate our expected utility from betting or passing,

$$\begin{aligned} u(P) &= P(AA|AJ)(-1) + P(AJ|AJ)(-b_2) \\ &\quad + P(JJ|AJ)(-b_3) \\ u(B) &= P(AA|AJ)(-2) + P(AJ|AJ)(1 - b_2) \\ &\quad + P(JJ|AJ)(4b_3 + (1 - b_3)) \end{aligned}$$

Since our opponent is more likely to hold AJ than AA – that is, $P(AJ|AJ) > P(AA|AJ)$ – a little arithmatic reveals that $u(B) > u(P)$ regardless of our opponent's choices for p_2, p_3. It follows that $p_2 = 1$.

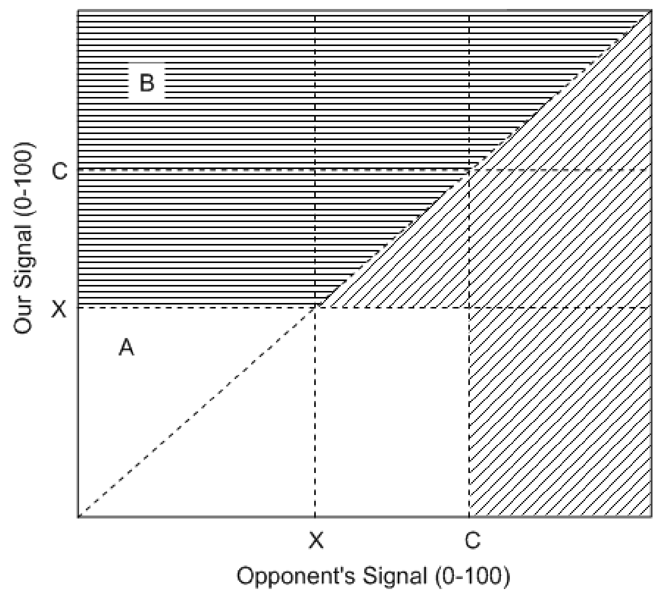
Finally, assume our opponent plays $(1, 1, p_3)$ and that we hold JJ. Here our utilities are

$$\begin{aligned}
u(P) &= P(AA|JJ)(-1) + P(AJ|JJ)(-1) \\
&\quad + P(JJ|JJ)(-b_3) \\
&= \frac{6}{15}(-1) + \frac{8}{15}(-1) + \frac{1}{15}(-b_3) \\
&= \frac{-14 - b_3}{15} \\
u(B) &= P(AA|JJ)(-2) + P(AJ|JJ)(-2) \\
&\quad + P(JJ|JJ)((1 - b_3) + 8b_3) \\
&= \frac{6}{15}(-2) + \frac{8}{15}(-2) + \frac{1}{15}(1 + 7b_3) \\
&= \frac{-27 + 7b_3}{15}
\end{aligned}$$

For any $b_3 \in [0, 1]$, passing is less costly than betting, so our best response is to always pass on JJ.

Thus, the only possible NE strategy is (1,1,0) – always betting AA, AJ; always folding JJ. Since (1,1,0) is a best response to itself, it is indeed a symmetric Bayesian NE.

7. (a) Passing with any signal above C is strongly dominated. If the opponent holds a lower signal, we gain by fighting; if he holds a higher signal, he will fight anyway. We can therefore assume the best response cutoff $X < C$.



In the graph above we depict the possible distributions of signals. In the unshaded region A, both players run, so our payout is 40. In the shaded region, one or both players fight. We win 100 in the area B above the 45° line, and 0 below. Our expected payout is:

$$\begin{aligned} u(X) &= 40P(A) + 100P(B) \\ &= \frac{40\text{Area}(A) + 100\text{Area}(B)}{\text{Total Area}} \\ &= \frac{40(CX) + 100(\frac{1}{2}(100^2 - X^2))}{100^2} \end{aligned}$$

Maximizing with respect to X, we find $X^* = .4C$, as desired.

(b) If both players use a best-response, then $C_1 = .4C_2$ and $C_2 = .4C_1$, so $C_1 = C_2 = 0$ and they always fight.

(c) The conditional expected utility of fighting is monotonic in the signal received. Expected utility is therefore maximized by a cut-off strategy with C such that $E(F|C) = E(R|C)$. Thus, one best response is this cutoff strategy.

Note, that it is possible for other strategies to give equal utility, and hence also be best responses. For example, if the opponent's strategy is to always fight, then any response yields the same utility.

(d) Since a NE consist of mututally best-response strategies, these strategies must be cutoff. By (b), each player uses a cutoff of zero, so they always fights.