1. If each robber has a $F(y)$ chance of being caught, then $NxF(y)$ of the Nx who try to steal are caught; the rest steal successfully. We assume this means that $NxF(y)$ cops catch one robber each, while the rest of the Ny sent to patrol return emptyhanded.

   Denote the collective utility functions $U_r(x, y)$ and $U_c(x, y)$. Then

$$\begin{aligned} U_r(x, y) &= N(1-x)(0) + NxF(y)(-5) + [Nx - NxF(y)](5) \\ &= 5Nx[1 - 2F(y)] \\ U_c(x, y) &= N(1-y)(1) + NxF(y)(5) + [Ny - NxF(y)](-1) \\ &= N[1 - 2y + 6xF(y)) \end{aligned}$$

   Note that it is possible that $Ny - NxF(y) < 0$, in which case the last term of U_c doesn't really make sense. We feel, however, that our model is the most faithful interpretation of the problem statement. One interpretation might be that when$Ny - NxF(y) < 0$, each cop has caught more than one robber each, so there is an additional utility bonus.

   It is easily verified that sending all or none of the employees cannot be an equilibrium strategy for either Fagin or Gordon. We are therefore looking for interior equilibria $(x, y \in (0, 1)$ so we require

$$\frac{\partial}{\partial y} U_c(x, y) = \frac{\partial}{\partial x} U_r(x, y) = 0$$

   Solving these FONCs yields

$$F(y) = \frac{1}{2} \quad \text{and} \quad F'(y) = \frac{1}{3x},$$

   which we can use to solve for x and y given any $F(y)$.

   (a) For $F(y) = y$, the utility functions are the same as the game discussed in class. As expected, this yields the same pure NE, $(x, y) = \left(\frac{1}{3}, \frac{1}{2}\right)$.

   (b) For $F(y) = y^2$, the FONC's are satisfied at $\left(\frac{1}{3\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$. However, $\frac{\partial^2 U_c}{\partial y^2} > 0$, so the cops' utility function is convex, and $y = 1/\sqrt{2}$ is a minimum. Thus, there is no NE in pure strategies.

(c) For $F(y) = \sqrt{y}$, the FONC are satisfied at $\left(\frac{1}{3}, \frac{1}{4}\right)$.